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The moving averages demystified

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Abstract

A common method in technical analysis is the construction of moving averages along time series of stock prices. We show that they present a practical interest for physicists, and raise new questions on fundamental ground. Indeed, self-affine signals characterized by a defined roughness exponent H can be investigated through moving averages. The density ρ of crossing points between two moving averages is shown to be a measure of long-range power-law correlations in a signal. Finally, we present a specific transform with which various structures in a signal, e.g. trends, cycles, noise, etc. can be investigated in a systematic way. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

In Physics, theories are mainly motivated by observations. Conversely, experiments are set up in order to either confirm or infirm a theory. In Finance, the situation is quite different: there is a huge gap between econometry and empirical finance. Indeed, the hypothesis of a pure random stock market is almost taken for granted in econometry while it is definetely not considered as such in empirical finance. Econophysicists are trying to fill the above gap as Stanley said in his contribution "Can Physics contribute to Finance?" [1].

The question of the present contribution is quite the opposite: "Can Finance contribute to Physics?". The answer of this question is undoubtedly YES. We will illustrate this answer in the particular case of a technical tool, the so-called moving averages.

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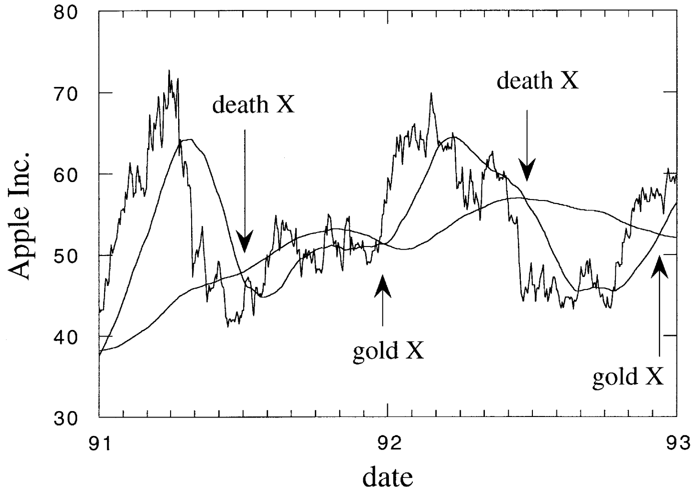

Fig. 1. Two moving averages $\bar{y}_1$ and $\bar{y}_2$ of a financial data series (Two year evolution of the Apple stock price) for $T_1 = 50$ and for $T_2 = 200$. Two "gold crosses" and a two "death crosses" are denoted (see definition in the main text).

Moving averages are common tools in Technical Analysis [2,3]. By definition, a moving average $\bar{y}$ at time t of a signal y is

$$\bar{y} = \frac{1}{T} \sum_{i=0}^{T-1} y(t-i) , \tag{1}$$

where T is the time interval over which the average is calculated. It is easy to show that if the trend of $y(t)$ is positive, the moving average $\bar{y}$ will be below y, while $\bar{y} > y$ when the trend is negative.

Consider two different moving averages $\bar{y}_1$ and $\bar{y}_2$ characterized, respectively, by T_1 and T_2 intervals such that $T_2 > T_1$. These moving averages are illustrated in Fig. 1 for the specific case of a typical financial time series, i.e. the evolution of Apple stock price from January 1st 1987 till December 31th 1996, and for the parameter values $T_1 = 50$ and $T_2 = 200$. The crossings of $\bar{y}_1$ and $\bar{y}_2$ coincide with drastic changes of the trend of $y(t)$. If $y(t)$ increases for a long period before decreasing rapidly, $\bar{y}_1$ will cross $\bar{y}_2$ from above. This event is called a "death cross" in empirical finance [2]. On the contrary, if $\bar{y}_1$ crosses $\bar{y}_2$ from below, the crossing point coincides with an upsurge of the signal $y(t)$. This event is called a "gold cross". Chartists often try to "extrapolate" the evolution of $\bar{y}_1$ and $\bar{y}_2$ expecting "gold" or "death" crosses. Most computers on trading places are equiped for performing this kind of analysis and forecasting [3]. Such curves are automatically displayed on stock charts on most trading softwares. Obviously, the positions of the crossing points are determined by the past history of the data and not to the future such that the forecasting in empirical finance as based on moving average "recipes" are far from a short-/long-range quantitative forecasting.

As physicists, we do not endorse these charting methods. Even though moving averages seem to be poor statistical measures, we will however see in this paper that they present some very practical interest for physicists and raise new questions in statistical physics.

2. Crossing points

The artificial time series used for the following demonstration within the successive random addition method originates in $d=1$ landscape profile construction. This method is also called “midpoint displacement” in the literature [4]. With this algorithm based on iterations, one generates a sequence of length $N = 2^n + 1$ where n is an iteration number. At each iteration, one finds the intermediate positions (midpoints) of couples of neighboring points and calculates the values of the signal at the midpoints through some interpolation with respect to neighboring couples. The midpoint values are then displaced by random numbers chosen from a normal distribution with zero mean and variance $\sigma^2/2^{2nH}$. The parameter H is the Hurst exponent of the resulting self-affine signal or fractional Brownian motion. For such a (discrete) self-affine signal $y(t)$, we can choose a particular point on the signal and rescale its neighborhood by a factor b using the roughness (or Hurst [5]) exponent H and defining the new signal $b^{-H}y(bt)$. For the correct exponent value H, the signal obtained should be indistinguishable from the original one, i.e. $y(t) \sim b^{-H}y(bt)$. The random walk corresponds to $H = \frac{1}{2}$.

We have built several artificial time series up to $N = 262145$ data points ($n = 18$ iterations) and for various values of H. For each signal, we have considered two moving averages $\bar{y}_1$ and $\bar{y}_2$ and have calculated the density ρ of crossing points for both moving averages $\bar{y}_1$ and $\bar{y}_2$ as a function of T_1 and T_2. In all checked cases, ρ is independent of the size N of the time series. In so doing, the fractal dimension of the set of crossing points is unity, i.e. the points are homogeneously distributed in time along $\bar{y}_1$ and $\bar{y}_2$.

Let us continue the analysis of moving averages. When the period T is large, $\bar{y}(t)$ is smooth and “relatively distant” from the signal $y(t)$ while for small T values, $\bar{y}(t)$ rather follows the excursion of the signal. Thus, it is of high interest to observe how ρ behaves and whether it has some scaling behavior with respect to the relative difference $0 < \Delta T < 1$ defined as $\Delta T = (T_2 - T_1)/T_2$.

Fig. 2 presents on linear scales the plot of ρ as a function of ΔT for $H = 0.3, 0.5$ and 0.7. The parameter T_2 was fixed to be 80. The $\rho(\Delta T)$ curve seems to be fully symmetric and diverges for $\Delta T = 0$ and for $\Delta T = 1$, i.e. for identical $\bar{y}_1$ and $\bar{y}_2$. For $T_1 = T_2/2$, the density of crossing points has a minimum. Moreover, for small ΔT values, we find that ρ scales as ΔT^{H-1} as well as $\rho \sim (1 - \Delta T)^{H-1}$ for ΔT values close to 1. We have also found that ρ scales as T_2^{-1}. Considering the above behaviors, we propose the general form for the density of crossing points

$$\rho \sim \frac{1}{T_2}[(\Delta T)(1 - \Delta T)]^{H-1} . \tag{2}$$

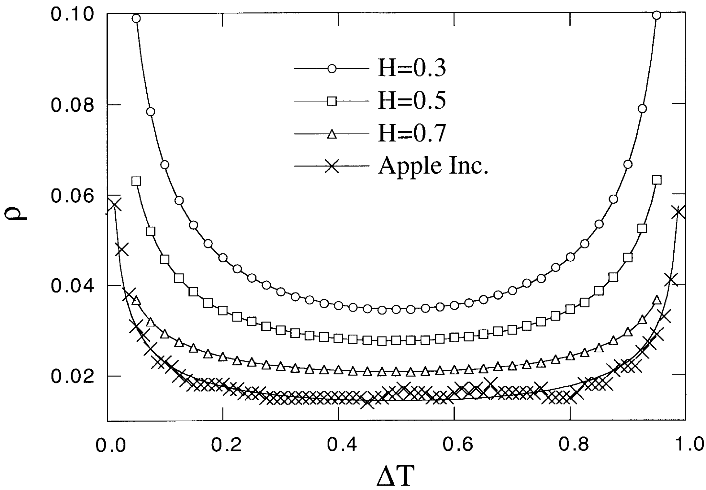

Fig. 2. The density ρ of crossing points as a function of the relative difference ΔT with $T_2 = 80$. Different values of H are illustrated: $H = 0.3$, 0.5 and 0.7. The case of the Apple stock price for a period of 10 years (January 1st 1987 till December 31th 1986) is also illustrated. Continuous curves are fits using Eq. (2).

Two time scales appear in Eq. (2): ΔT and T_2. Thus, the time difference ΔT allows to hold a parameter for investigating the correlations (H) lying in the signal. The largest period T_2 controls trivially the amplitude of ρ: the biggest is T_2, the smoothest is the mobile average $\bar{y}_2$ and the least is the number of crossing points. The continuous curves in Fig. 2 represent a fit of the data using Eq. (2). The agreement of our conjecture (2) and the data is quite remarkable. It has been shown in Ref. [6] that the computation of moving averages provides a very accurate measure of the roughness exponent H. In fact, the measure of H is as accurate as that provided by the detrended fluctuation analysis (DFA) [7].

Two extreme cases can be discussed: (i) $\Delta T \approx 0$ and (ii) $\Delta T \approx 1$. The former situation corresponds to $T_1 = T_2 - 1$ and the difference between both moving averages is then given by

$$\bar{y}_2 - \bar{y}_1 = \frac{1}{T_2}[y(t - T_2 + 1) - \bar{y}_1] \tag{3}$$

such that the existence of a crossing point at time t is determined by the crossing between the signal at time $t - T_2 + 1$ and the moving average $\bar{y}_1 \approx \bar{y}_2$. The latter situation corresponds to $T_1 = 1$, i.e. $\bar{y}_1 = y$ such that the existence of a crossing point at time t corresponds to a crossing between y and $\bar{y}_2$ at time t. One understands that the number of crossing points is roughly the same in both extreme cases. This argument explains also the symmetry of the density plot. In between both extreme cases, the situation is more complex and cannot be easily discussed herein.

Fig. 2 presents also the density plot for the Apple stock prices (data of Fig. 1). Again, a symmetric curve is obtained for ρ. A fit using Eq. (2) gives an estimation

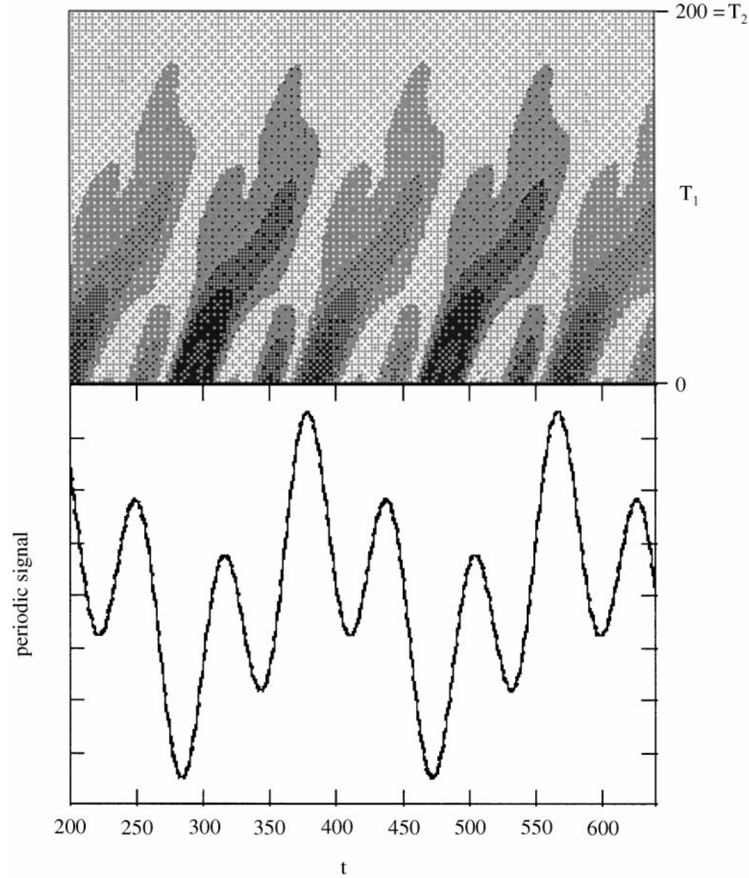

Fig. 3. An artificial signal taken here to be a sum of two sinusoids having two different frequencies and arbitrary phases (bottom) and the corresponding spectrum of moving averages (top). The long-term period T_2 has been fixed to 200. The y-axis of the top figure corresponds to T_1. Grey levels are a code for the distance between both moving averages.

for the Hurst exponent $H = 0.46 \pm 0.02$ in agreement with the values obtained using DFA $H = 0.47 \pm 0.03$ (not shown here due to the lack of space).

3. Spectrum of moving averages

It is easy to show that the distance from the signal y to the moving average $\bar{y}$ is proportional to the slope of the signal and proportional to the interval T. One such moving average is thus extracting some information about the *trend* of y over T. On this basis, we have developed a method that uses a larger set of moving averages in order to visualize different trends on any time scale.

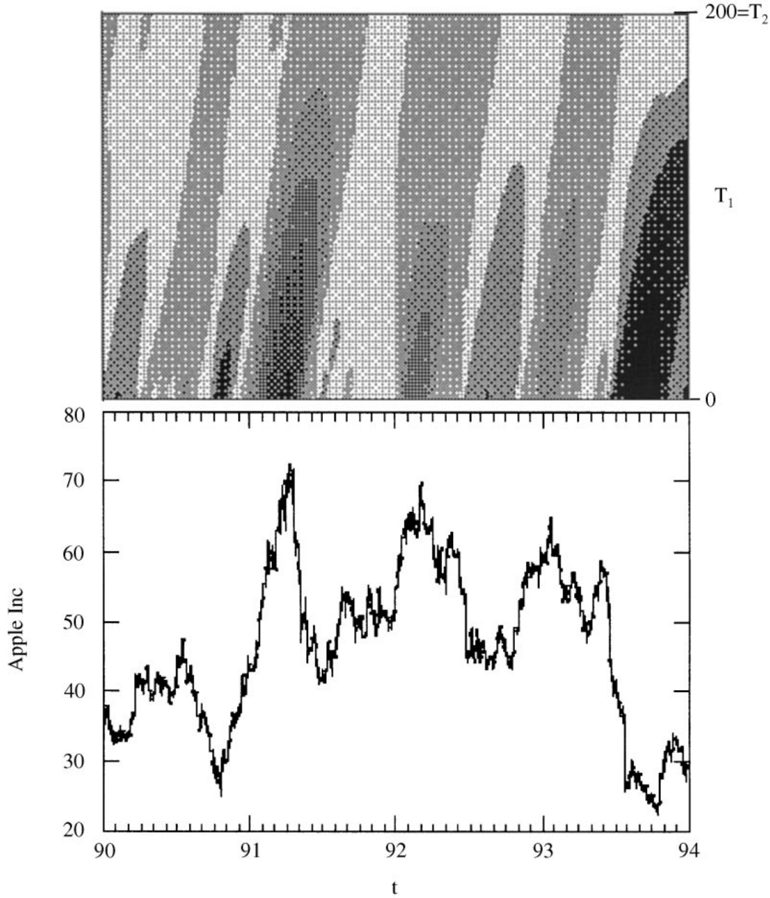

Fig. 4. Apple shares between 1990 and 1994 (bottom) and the corresponding spectrum of moving averages (top). The long term period T_2 has been fixed to 200. The y-axis of the top figure corresponds to T_1. Grey levels are a code for the distance between both moving averages.

The basic idea of the "spectrum of moving averages" is to fix the long-term period T_2 to a high value. Then, the short-term period T_1 is varied between 1 and $T_2 - 1$. The relative distance $\delta = (\bar{y}_1 - \bar{y}_2)/\bar{y}_1$ between both moving averages is then computed at each time step t and for all T_1 values. Fig. 3 presents the resulting pattern together with the evolution of an artificial signal taken here to be a sum of two sinusoids having two different frequencies and arbitrary phases. T_2 is here fixed to be 200. The grey levels describe the distance δ.[1] It should be noticed that the resulting pattern has also a periodic structure. Two different frequencies can be easily distinguished in distinct parts of the spectrum as predicted by Eq. (2). Moreover, it should be seen that the crossing of the moving averages ($\delta = 0$) corresponds to inclined curves crossing the

[1] Color pictures are available on the web site: http://www.supras.phys.ulg.ac.be/statphys/statphys.html.

pattern perpendicular to the t direction as observed in Fig. 3. Thus, the method allows for visualizing cycles and trends. This is quite useful in finance but also in physics.

A typical financial pattern is the evolution of the Apple stock price between 1990 and 1994. It is illustrated in Fig. 4. A periodic-like structure is suggested to occur in the spectrum of moving averages. Indeed, an alternance of positive and negative δ's are found in the spectrum parallel to the time axis. This case is only shown for illustrative purpose, deeper analysis will be done in a near future but our example (Fig. 4) should convince the reader about the interest of this visualization technique.

4. Conclusion

We have shown that a poor statistical tool though very commonly used in empirical finance can contribute to fundamental physics. Fractional Brownian motions have been considered. They lead to a non-trivial density of crossing points of moving averages. This has been used to develop some analysis techniques. A spectral transform has been shown to provide a useful technique for visualizing the trends and cycles on various length scales lying in such a signal.

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