ABSTRACT
Notwithstanding it’s widespread use and well documented profitability, technical analysis is still not accepted by the academia. This may be due to the fact that the causes of it’s profitability are not yet identified. To close the gap, this paper suggests a rationale for the application of technical trading rules. It is shown, that a moving average rule can be interpreted as a cheap proxy for Bayesian learning. Within a realistic environment the trading rule is able to detect possible regime shifts in the process of the underlying fundamentals and cannot be attributed as irrational.

JEL classification: F31, F37; C32, G12, G15
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1. Introduction

The term “technical analysis” in general contains a large variety of trading techniques, which are based on past movements of the asset price and a few other related variables. The use of trading rules to detect patterns in the time series of asset prices dates back to the 1800s. In contrast to the asset markets today, a trader was clearly not able to develop a fundamental analysis on the basis of extensive financial information. Persistent shifts in supply and demand had to be detected in past price movements by using techniques that range from simple to quite elaborate. Many of these techniques are still used by practitioners as is documented in studies of micro survey data. The attitude of the academia towards technical analysis is at least reserved, which is due to the believe of economists that financial markets in particular are well described by the efficient market hypothesis. Under these circumstances it is obvious that trading rules, generally not derived from a mathematically well defined econometric or economic model, should not be very informative. This view was seriously challenged by a large body of empirical studies showing that on the one hand technical trading rules are indeed successful,1 and on the other hand standard martingale models do not sufficiently describe short run price movements (Lewis, 1995 and Taylor, 1995).

The contribution of this paper is to provide a rationale for the application of technical trading rules by means of a simple illustration of the foreign exchange market. In contrast to the efficient market hypothesis, agents normally have incomplete knowledge about the true set of fundamental variables driving the exchange rate and information are available only with considerable lags. Lewis (1989) concludes that an appropriate exchange rate model should cover these issues by introducing learning processes. In these models developed so far, purely forward looking agents infer a possible regime shift only from subsequent observations of the fundamental under consideration. But if the exchange rate was indeed driven by this fundamental, which is not yet observable, useful information about the possible regime shift should be available by analysing past foreign exchange prices. In this paper it is shown, that within such a realistic informational environment, filter rules can be interpreted as a proxy for Bayesian learning. They are able to detect possible regime shifts in the underlying fundamentals and cannot be attributed as irrational. Conversely, pure forward looking

1 Especially the profitability of moving average rules are repeatedly reported in the literature. For exchange rates this is done recently by Neely et al. (1997), LeBaron (1999) and Lee et al. (2001), for stocks Brock et al. (1992).
expectations ignoring actual price movements, are irrational in the sense that they don’t make use of all currently available information. From this point of view, it seems to be clear, why in the days of Charles Dow technical analysis was the dominant forecasting strategy. Some empirical support for this interpretation of technical trading is provided by Markov regime switching models, where the latent regime variable follows a discrete state-space, first order Markov process and the regime probabilities are obtained from a Baysian filter procedure (Hamilton, 1989). In particular, Dewachter (1997) shows that the moving average rule predicts the latent variable remarkably well, if the exchange rate can be described by the segmented trends model of Engel and Hamilton (1990).

2. Learning from past exchange rates – a simple illustration

In the standard asset market approach in exchange rate economics the (log of the) exchange rate $e_t$ is driven by the expected rate of depreciation $E_t[\Delta e_{t+1}]$ and a set of contemporaneous fundamentals included in a vector $y_t$. Thus the exchange rate can be written as

$$e_t = by_t + \gamma E_t[\Delta e_{t+1}]$$

(1)

where the vector of elasticities of the contemporaneous variables $b$ and the elasticity of exchange rate expectation $\gamma$ should be constant over time. For the sake of simplicity the vector $y$ consists only of two uncorrelated variables $z$ and $n$ with their coefficients denoted by $\alpha$ and $\beta$, respectively:

$$e_t = \alpha z_t + \beta n_t + \gamma E_t[\Delta e_{t+1}]$$

(1’)

Under the rational expectations hypothesis equation (1) has the well known forward looking solution:
\[ e_t = \frac{\alpha}{1+\gamma} \sum_{j=0}^{\infty} \left( \frac{\gamma}{1+\gamma} \right)^j E_t [z_{t+j}] + \frac{\beta}{1+\gamma} \sum_{j=0}^{\infty} \left( \frac{\gamma}{1+\gamma} \right)^j E_t [n_{t+j}] \]

\[ = \frac{\alpha}{1+\gamma} \sum_{j=0}^{\infty} \left( \frac{\gamma}{1+\gamma} \right)^j E_t [z_{t+j}] + t_{t+N_t} \]

where \( t_{t+N_t} \equiv \frac{\beta}{1+\gamma} \sum_{j=0}^{\infty} \left( \frac{\gamma}{1+\gamma} \right)^j E_t [n_{t+j}] \).

Following Lewis (1989), we assume that \( n \) and \( z \) are stationary, but at time \( \tau \) a regime shift happens in the mean of the fundamental \( z \). Due to an announcement or an event, agents believe that the regime shift is possible, but are not sure, that it in fact occurs at time \( \tau \). For example, the central bank may decide to conduct a monetary policy that is suitable to bring down inflation and announces this regime shift at time \( \tau \). Nevertheless, private agents cannot be fully convinced of the announcement implying that they still assign a positive probability to the possibility of ongoing high inflation monetary policy. Assuming that the market knows the true values of the process before and after the switch, \( z_t \) evolves according to

\[ z_t = \tilde{\mu} + \tilde{\vartheta}, \quad \text{with} \begin{cases} \tilde{\mu} = \mu_0, & \text{for } t < \tau \\ \tilde{\mu} = \mu_1, & \text{for } t \geq \tau \end{cases} \]

where \( \mu_0 < \mu_1 \) and \( \tilde{\vartheta} \sim N(0, \sigma^2) \). As long as \( t < \tau \), the exchange rate can easily be calculated by taking expectations of (3) and substituting the result into equation (2):

\[ e_t = \alpha \mu_0 + \alpha \tilde{\vartheta} + t_{t+N_t}, \quad \text{for } t < \tau \]

At time \( \tau \), the market is uncertain about the true process of the fundamental \( z \). Of course, the uncertainty will be reduced as agents make observations about \( z \) over time and discover whether \( \mu_0 \) or \( \mu_1 \) is the true mean of the process. The reduction of uncertainty can be characterised by a learning process, where market participants generate Bayesian forecasts and assign a probability to either process. Defining \( P_{\mu_j} \) as the probability that the process at \( t > \tau \) is driven by \( \mu_j \), the market’s expectation of \( z_t \) is

\[ E_{t-1}[z_{t+j}] = P_{0,t-j}\mu_0 + P_{1,t-j}\mu_1, \quad \text{for all } t > \tau \]
The Probabilities $P_{j,t}$ are updated with every realisation of $z$ according to Bayes’ law:

$$P_{j,t} = \frac{P_{j,t-1} \cdot f_j(z_t | \bar{\mu} = \mu_j)}{\sum_{j=0}^{1} P_{j,t-1} \cdot f_j(z_t | \bar{\mu} = \mu_j)}, \quad j = 0, 1 \text{ and for all } t > \tau$$

(6)

where $f_j(z_t | \bar{\mu} = \mu_j)$ is the density function of $z_t$, if $\mu_j$ is the true mean of the process. The odds at time $t$ that the process has switched is

$$\frac{P_{1,t}}{P_{0,t}} = \frac{P_{1,t-1} \cdot f_1(z_t | \bar{\mu} = \mu_1)}{P_{0,t-1} \cdot f_0(z_t | \bar{\mu} = \mu_0)}, \quad \text{for all } t > \tau$$

(7)

Taking logs and solving the resulting difference equation yields

$$\log \left( \frac{P_{1,t}}{P_{0,t}} \right) = \log \left( \frac{P_{1,\tau}}{P_{0,\tau}} \right) + \sum_{j=\tau}^{1} \left( \frac{\mu_0^2 - \mu_j^2}{2\sigma^2} + 2(\mu_1 - \mu_0)z_j \right).$$

(8)

Equation (9) can be solved for the probabilities $P_{j,t}$ using the fact that $P_{0,t} = 1 - P_{1,t}$:

$$P_{1,t} = \frac{\exp \left[ \sum_{j=\tau}^{1} \left( \frac{\mu_0^2 - \mu_j^2}{2\sigma^2} + 2(\mu_1 - \mu_0)z_j \right) \right]}{1 + \exp \left[ \sum_{j=\tau}^{1} \left( \frac{\mu_0^2 - \mu_j^2}{2\sigma^2} + 2(\mu_1 - \mu_0)z_j \right) \right]}$$

(10.1)

2 This is just to simplify calculations.
and

\[ P_{0,t} = \frac{1}{1 + \exp \left( \sum_{j=\tau}^{t} \left( \frac{\mu_0^2 - \mu_1^2}{2} + 2(\mu_1 - \mu_0) z_j \right) \right) \frac{1}{2\sigma^2}} \]  

(10.2)

Together with (2) and (5) the exchange rate at \( t > \tau \) is

\[ e_t = \alpha(P_{0,t}\mu_0 + P_{1,t}\mu_1 + \vartheta_t) + \epsilon_t N_t, \quad \text{for} \ t > \tau \]  

(11)

From equations (10) and (11) the central results of Lewis (1989) become obvious. After the regime shift, the exchange rate is driven by the probabilities assigned to either process, which move over time in response to new information regarding the fundamental \( z \). If in fact there has been a regime shift from \( \mu_0 \) to \( \mu_1 \), the probability \( P_{1,t} \) converges to one (plim \( P_{1,t} = 1 \)) and

\[ \lim_{t \to \infty} e_t = \alpha(\mu_1 + \vartheta_t) + \epsilon_t N_t \]  

(12)

The convergence to the new equilibrium exchange rate is slow, if - ceteris paribus - the regime shift is small or the variance of the \( z_t \) is high, because a given observation cannot easily be assigned to one of the possible regimes and small changes in the probabilities result in small changes of the exchange rate. Conditional on the information available at time \( t \), the expectations of market participants are rational in that they minimize forecast errors. Clearly, based on the ex post available knowledge of the true stochastic process \( z_t \), the forecasting errors are biased over the whole learning process.

In the following, we show why technical analysis may work in the characterised informational environment. On the basis of the learning process we calculate optimal trading decisions for a single trader, who’s excess demand for foreign currency is not large enough to change prices.

\[ \text{Note that the exchange rate jumps at } t = \tau, \text{because the market’s uncertainty results in a sudden change of the regime probabilities.} \]
on the foreign exchange market. If $\mu_0 < \mu_1$ implies an exchange rate increase, the trader buys foreign currency at time $t > \tau$, if $P_{1,t} > P_{0,t}$. With (9) we can reformulate this condition to

$$
\text{Buy, iff } \frac{1}{t-\tau} \sum_{j=\tau}^{t} (\mu_0^2 - \mu_1^2) + 2(\mu_1 - \mu_0)z_j > 2\sigma^2 > 0.
$$

or

$$
\text{buy, iff } \frac{1}{t-\tau} \sum_{j=\tau}^{t} z_j > \frac{\mu_0 + \mu_1}{2}.
$$

According to (13) a buy signal is given at time $t$, whenever the average of the $t-\tau$ observations of $z$ exceeds the average of the two parameters $\mu_0$ and $\mu_1$.

The results were derived under the assumption that collecting and processing information about fundamentals is costless. The only problem arises from the identification of the true parameter ($\mu_0$ or $\mu_1$) driving the fundamental variable $z$. As described briefly in the introduction, we now drop this simplifying assumption to make the informational environment of the foreign exchange trader more realistic. Suppose again that the market has recognized the possibility of the regime shift at time $\tau$, and again one regime is perceived to be as likely than the other ($P_{1,\tau} = P_{0,\tau}$). In contrast to the above analysis, information is costly and observations about $z$ are available only with a considerable lag of $\delta$ periods. At time $\tau$, the exchange rate responds to the possibility that a regime shift has happened. In the following $\delta$ periods, market’s forward looking expectations remain fixed, because there are no observations of the fundamental available until time $\tau + \delta$. Exchange rate changes are due to contemporaneous realisations of $z$ and $n$ alone:

$$
e_t = \frac{\alpha}{1 + \gamma} (\mu_1 + \hat{\theta}_t) + \frac{\alpha\gamma}{1 + \gamma} (P_{0,\tau} \cdot \mu_0 + P_{1,\tau} \cdot \mu_1) + \gamma N_t, \text{ for } \tau < t < \tau + \delta \quad (14)
$$

In contrast to this “wait and see-strategy”, technical analysis tries to infer information about $z$ through past movements of the exchange rate. To show how this might work, we construct a trading rule for time $\tau + \kappa$ on the Bayesian logic of (13):

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4 If information costs are sufficiently high, no observations are available at all.
Buy, iff \( \frac{1}{\kappa} \sum_{j=\tau}^{\kappa+\gamma} e_j > \frac{E_t\{e_t|\mu=\mu_0\} + E_t\{e_t|\mu=\mu_1\}}{2}, \kappa = 1,\ldots,\delta \) \hspace{1cm} (15)

The trading rule (15) can be proven by taking expectations using (14):

\[
E_{t+\kappa}\left[\frac{1}{\kappa} \sum_{j=\tau}^{\kappa+\gamma} e_j\right] = \frac{\alpha}{1+\gamma} \frac{\mu_1 - \mu_0}{2} + \alpha \frac{\mu_0 + \mu_1}{2} + \bar{N}, \kappa = 1,\ldots,\delta \hspace{1cm} (16)
\]

and

\[
E_{t+\kappa}\left[E_t\{e_t|\mu=\mu_0\} + E_t\{e_t|\mu=\mu_1\}\right] = \alpha \frac{\mu_0 + \mu_1}{2} + \bar{N}, \kappa = 1,\ldots,\delta \hspace{1cm} (17)
\]

where \( \bar{N} \) is the long run impact of the fundamental \( n \) on the exchange rate. With a rising number of exchange rate observations (\( \kappa \)) the impact of both fundamental’s innovations cancels out, so that the difference between (16) and (17) converges to \( \frac{\alpha}{1+\gamma} \frac{\mu_1 - \mu_0}{2} \), i.e. the impact of contemporaneous realisation of \( z \) on the exchange rate. If in contrast, the regime shift has not occurred, the difference is zero and a buy signal is not given. This implies, that past movements of the exchange rate are able to provide useful information about the process of the underlying fundamentals, which can be filtered by means of a simple average rule. Conversely, purely forward looking agents ignoring actual price movements, are irrational in the sense that they don’t make use of all currently available information.

The analysis can be generalized in different ways. The assumption that observations of \( z \) are available with a lag of \( \delta \) periods includes that some fundamentals are not observable at all (e.g. tastes and preferences). In this case, \( \delta \) reaches infinity and market participants can only infer information about \( z \) through exchange rates. Another generalisation may result from the assumption that the time \( \tau \), at which the possible regime switch occurs, is not common knowledge on the foreign exchange market. An uninformed trader may compensate his lack of information using a daily updated average rule (a moving average rule) to permanently check exchange rates for potential shocks. Besides these informational asymmetries, this moving
average trading rule may as well be applied to detect regime shifts, which are not directly observable and are not revealed by an announcement.

Although this strategy seems to be suitable, the practical implementation of the moving average rule is not straight forward. First, to make use of a filter rule like (15) it is necessary to find an unbiased approximation for \( E_t[e^0|\mu = \mu_0] + E_t[e^1|\mu = \mu_1] \). A convenient solution to this problem is base on the fact, that the exchange rate at times \( t < \tau \) follows (4) implying \( \tilde{\mu} = \mu_0 \) and at times \( t \geq \tau \) follows (14) implying \( \tilde{\mu} = \mu_1 \). If the market’s probability assigned to the regime shift is close to one then the average of the last \( \psi = 2\delta \) observations is an unbiased approximation for (17). However, because the market is uncertain about the regime shift, \( P_{1,\tau} \) is significantly lower than one, so that the span of the average has to be smaller:

\[
\psi = \delta \cdot \frac{2(1 + \gamma \cdot P_{1,\tau})}{1 + \gamma} \tag{18}
\]

Therefore, the trading rule (15) can be reformulated to

\[
\text{Buy, iff } \frac{1}{\delta} \sum_{j=\tau}^{\tau+\delta} e_j > \frac{1}{\psi} \sum_{j=\tau+\delta-\psi}^{\tau+\delta} e_j \tag{19}
\]

Second, the stochastic process of the exchange rate may not be stationary as is assumed in the above analysis. For example, there may be long swings in the time series of the exchange rate, as is stated in Engel and Hamilton (1990). In this case, the exchange rate probably follows a stable upward or downward trend, but the trading rule (19) would wrongly detect a regime shift of on of the underlying fundamentals. To avoid these serious misinterpretations, the trader has to use first differences of the exchange rate implying that the moving average rule now reveal some information about the turning point in the time series. Third, information about the date of the possible regime shift is not available, the span of both the long and the short moving average cannot be exactly determined. So arbitrarily chosen values for \( \psi \) and \( \delta \) will produce biased results, but not permit the trading rule to give adverse signals. Forth and most important, the above results are derived from an incomplete exchange rate model, because the market doesn’t really make use of the filter rule. But due to it’s reinforcing nature, an initially given buy signal would result in an ongoing appreciation of the exchange rate.
Consequently, an exchange rate model capturing these features must exhibit some mean reversion properties as well.

3. Concluding remarks

This paper suggests a rationale for the application of technical trading rules. In contrast to the assumptions of the efficient market hypothesis, market participants normally have incomplete knowledge of the true set of fundamental variables driving the exchange rate and information about these variables are available only with considerable lags. It is shown that technical analysis can be interpreted as a kind of cheap Baysian learning. Especially, within a realistic informational environment, a moving average rule is able to detect possible regime shifts in the underlying fundamentals and cannot be attributed as irrational. Pure forward looking expectations ignoring recent price movements, are irrational in the sense that they don’t make use of all currently available information. Clearly, the results are based on the assumption that the market doesn’t make use of the filter rule. But if market participants really trade on the rule, an initially given buy signal would result - due to it’s reinforcing nature - in an ongoing appreciation of the exchange rate. Consequently, an exchange rate model capturing these features must exhibit some mean reversion properties as well. This is beyond the scope of the paper and is left for further research.
Literature


