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The predictability of asset returns: an approach combining technical analysis and time series forecasts

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Abstract

We investigate predictability of asset returns by developing an approach that combines technical analysis and conventional time series forecasts. While exploiting predictable components as functions of past prices or returns, technical trading rules and time series forecasts capture different aspects of market predictability: the former tends to identify periods to be in the market when returns are positive and the latter is capable of identifying periods to be out when returns are negative. Applied to daily Dow Jones Averages over the first 100 years, the combined strategies outperform both technical trading rules and time series forecasts. The predictability can be explained largely by non-trivial low-order serial correlations in returns and is not mainly attributable to measurement errors arising from non-synchronous trading.

Keywords: AR-GARCH models; Combining forecasts; Excess returns; Predictability; Technical trading rules

1. Introduction

Various empirical studies have documented substantial evidence on the predictability of asset returns (Fama & French, 1988; Lo & MacKinlay, 1988; Poterba & Summers, 1988; Brock, Lakonishok, & LeBaron, 1992; Chan, Jegadeesh, & Lakonishok, 1996; Bessembinder & Chan, 1998; Allen & Karjalainen, 1999; Lo, Mamaysky, & Wang, 2000). Their studies suggest that asset returns are correlated, and hence, predictability can be captured, at least to some degree, by technical trading rules or by certain time series models. The primary purpose of this paper is to demonstrate how one can develop trading strategies which combine technical analysis and time series forecasts.

This paper differentiates itself from previous studies in the literature in the following aspects. Most empirical work has studied technical trading rules and time series models—in isolation. This is ultimately not satisfactory because, as shown in this study, technical trading rules

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and time series models are able to identify different predictable components. We develop an approach combining technical analysis and time series forecasts and apply it to Dow Jones Averages. Unlike most of the previous studies which focused only on the Dow Jones Industrial Average, we study all three Dow Jones Indexes (namely Industrial, Transportation and Utilities Averages) and apply exactly the same trading rules to them to test the robustness of our findings. In addition, a more extended sample period in comparison with those considered in previous studies (e.g. Brock et al., 1992; Bessembinder & Chan, 1998) is covered. The use of an expanded dataset is important since repeated visits of the same dataset could lay to suspicion of data snooping. Furthermore, we use rolling techniques to generate out-of-sample forecasts to guard against overfitting in time series modeling and avoid a potential bias induced by ex post selection.

Our main findings are as follows. Firstly, trading strategies combining technical analysis and time series forecasts are superior to either technical trading rules or time series forecasts. The fact that technical trading rules and time series forecasts are asymmetric in the opposite directions during buy and sell periods: the former tends to identify periods to be in the market when returns are positive and the latter is capable of identifying periods to be out when returns are negative, provides a striking evidence of their complementary properties. The combined trading strategies yield higher returns during both buy and sell periods and require fewer transactions than either technical trading rules or time series models. To examine the possibility of excess returns after accounting for transaction costs, we calculate the break-even costs for all trading strategies considered. For three Dow Averages over the full sample period, the average break-even costs for technical trading rules are between 0.6 and 0.8%. Those for time series models are small (less than 0.3%) due to the large number of transactions required. In all the cases considered, we found that the break-even costs for combined strategies, ranging from about 1.0% to more than 1.9%, are always greater than those from the corresponding technical trading rules.

Secondly, the robustness of the results across various subperiods supports our conclusions drawn from the full sample. Consistent with many earlier studies which documented that the forecasting ability of technical trading rules declined over the past decades, the predictability in returns from the combined trading strategies also deteriorates in recent years. For example, the average break-even costs of combined trading strategies for the Industrial Average in the subperiods from 1921 to 1969 are in excess of 1.2%, higher than the 1.0% level after 1970.

Thirdly, our results suggest that across the trading strategies examined, the Industrial Average is the most difficult to predict, as indicated by low levels in both excess returns over the buy-and-hold (pre-trading cost) and break-even costs. Over the sample period considered there seems no obvious difference in break-even costs between the Transportation and Utilities Averages.

Finally, since there is growing consensus among financial economists that non-synchronous trading induces spurious serial dependence in index returns (Scholes & Williams, 1977; Lo & MacKinlay, 1990), we investigate the sensitivity of returns to implementation of a 1-day lag, in which trading returns are measured beginning 1 day after a trading signal is initiated. Overall, the break-even costs for both technical trading and combined strategies reduce slightly in percentage after the non-synchronous adjustment, especially during subperiods before 1971. Therefore, the forecasting ability is not mainly attributable to return measurement errors arising from non-synchronous trading.

The paper is organized as follows. Section 2 is devoted to some preliminary analysis of the data. Section 3 studies technical trading rules and forecasts based on conventional time series models. Section 4 contains analysis of combined trading strategies, and addresses the robustness of the results. We summarize and conclude in Section 5.

2. Data description and preliminary analysis

As three of the most influential indicators in the US stock market, the Dow Jones Averages are commonly used not only to assess the state of the economy but also to serve as the basis of some investable products.[1] The data used in this study are three daily Dow Jones Averages which are price-weighted averages based on daily closing values. They are provided by Dow Jones & Company, Inc. There is a 100-year record of the daily Dow Jones Industrial Average from May 26, 1896 to May 24, 1996. However, less data are available in the Transportation Average, from October 26, 1896 (the Railroads Average only to January 2, 1970). The record of Utilities Average is even shorter, from January 2, 1929. There were no data recorded for certain dates because of various market activities. Prior to June 1952, trading took place 6 days a week, but Saturday trading was discontinued afterwards. There was no Saturday trading in the summers during the late 1940s and the Exchange was closed due to World War I between August 2 and December 12, 1914. Of course, the market was closed on all national holidays.

The summary statistics of the data are presented in Table 1. From panel 1A, we see that means of (continuously compounded) returns for the Industrial and the Transportation Averages are 0.0179% and 0.0138% per trading day, or 4.47% and 3.45% per year, respectively. The mean return of the Utilities Average is only about 0.0051% per trading day or 1.27% annually, a sharp decline compared to those of the Industrial and Transportation Averages. The standard deviations of all three Averages are approximately of the same magnitude (about 1.1% daily or 27.5% annually), and they all show signs of skewness and heavy tails. Panel 1B reports the numbers of days with returns exceeding various bands ranging from 0.1 to 10%. In general, there are more days with positive returns than those with negative returns. About 80% of daily absolute returns are within the 0.1% band during the first 100 years. Most of the returns below the −10% mark were recorded during the 1929 market crash and the Great Depression (1929–1939), except for 3 days with returns less than −10% in October 1987 (one for the Industrial Average on October 19, 1987 and two for the Transportation Average on October 19 and 26, 1987). From this respect, the 10-year Great Depression was the most volatile period in the first 100-year history of the US stock market.

Selected autocorrelations up to lag 50 for the three Averages are reported in panel 1C. Some low-order autocorrelations are significant at the 5% level. The first-order autocorrelations for the three Averages are all positive and have the largest values within each series: 0.044, 0.110 and 0.082, respectively.

3. Technical trading rules and autoregressive models

3.1. Technical trading rules

We focus on the widely used double cross-

[1]Futures and options on the Industrial Average have been traded on the Chicago Board Options Exchange and the Chicago Board of Trading since October 6, 1997. Also, on January 20, 1998, the American Stock Exchange began trading shares in a fund that holds stocks in the Dow Jones Industrial Average.

Table 1
Summary statistics for daily returns

Panel A	Industrial	Transportation	Utilities
N	27,597	27,469	17,954
Mean	0.000179	0.000138	0.000051
Std.	0.010872	0.011619	0.011063
Maximum	0.142729	0.164597	0.166246
Minimum	−0.279594	−0.192361	−0.186128
Skewness	−1.209782	−0.103829	−0.253528
Kurtosis	38.253490	16.214420	27.874250

Panel B No. of obs. with returns	Industrial	Transportation	Utilities
>0.0	14,425	13,850	9032
<0.0	13,009	13,368	8540
>0.001	12,917	12,376	7852
<−0.001	11,448	11,983	7359
>0.01	3122	3357	1551
<−0.01	3044	3180	1571
>0.10	4	10	6
<−0.10	5	6	4

Panel C Autocorrelations Lag	Industrial	Transportation	Utilities
1	0.044*	0.110*	0.082*
2	−0.027*	0.010	−0.022*
3	0.013	0.013	0.002
4	0.038*	0.023*	0.054*
5	0.027*	0.031*	0.028*
6	−0.020*	−0.013	−0.027*
7	−0.022*	−0.001	−0.015
8	0.015	0.010	0.032*
9	0.014	0.009	0.026*
10	0.011	0.014	0.044*
20	0.011	0.014	0.025*
30	0.021*	0.018*	0.011
40	0.003	0.001	0.003
50	0.009	−0.004	0.010

Results are presented for the sample period from May 24, 1896 to May 24, 1996 for the Industrial Average, from October 26, 1896 to May 24, 1996 for the Transportation Average, and from January 2, 1929 to May 24, 1996 for the Utilities Average. Skewness and Kurtosis are computed from $n^{-1}\sum(y_t-\bar{y})^3/((n-1)^{-1}\sum(y_t-\bar{y})^2)^{3/2}$ and $n^{-1}\sum(y_t-\bar{y})^4/((n-1)^{-1}\sum(y_t-\bar{y})^2)^2$, respectively. Autocorrelations (ρ) marked by asterisk in panel C indicate the test H_0: $\rho=0$ is significant at 5%.

over trading strategies in which two moving averages are calculated and trading signals are generated when the two moving averages intersect. These trading rules are typical trend-following methods and serve frequently as the basis for more sophisticated schemes. Throughout this paper, we use $\{x_t\}$ to denote the price series and $\{y_t\}$ the continuously compounded returns, where $y_t \equiv \log x_t - \log x_{t-1}$.

3.1.1. Technical trading rules based on moving averages

Definition 1. A moving average is a linear transformation which can be written as a finite polynomial in the back shift operator L with non-time-varying parameters $\{\theta_i\}$:

$$M = \sum_{i=1}^{m} \theta_i L^i, \tag{1}$$

where $Lx_t = x_{t-1}$.

The series $\{M[x_t]\}$ is smoother than $\{x_t\}$ and can be used as a primary estimate of the market trend. The technical trading rule (m_1, m_2, d), determined by the band d and the two moving averages M_1 and M_2 with lengths $m_2 > m_1 \geq 1$, respectively, is defined as follows.

Definition 2. Buy signals are generated sequentially at the times $\{\tau_i^B, i \geq 1\}$, where

$$\tau_i^B \equiv \inf\{t : t > \tau_{i-1}^B, M_1[x_t] - M_2[x_t] > dx_{t-1}\}, \tag{2}$$

and sell signals are generated sequentially at the times $\{\tau_i^S, i \geq 1\}$, where

$$\tau_i^S \equiv \inf\{t : t > \tau_{i-1}^S, M_2[x_t] - M_1[x_t] > dx_{t-1}\}. \tag{3}$$

The initials τ_0^B and τ_0^S are defined as zero and the band d is non-negative.

In other words, (2) and (3) classify all days into buy, sell or no action. When the short moving average cuts above the long moving average from below by a percentage change larger than the band d, a buy signal is generated. On the other hand, if the short moving average falls below the long moving average from above by a percentage change larger than d, a sell signal is given. No signal is generated if the short moving average is inside the band. The band d is designed to reduce the number of trades caused by frequent whipsaws in the price series during non-trending markets.

The ranges of three parameters m_1, m_2 and d can vary greatly in practice, depending primarily on market volatility, the trader's time frame of investment and the history of the moving averages used. In this study, the 10 trading rules analyzed in Brock et al. (1992):[2] (1, 50, 0), (1, 50, 0.01), (1, 150, 0), (1, 150, 0.01), (5, 150, 0), (5, 150, 0.01), (1, 200, 0), (1, 200, 0.01), (2, 200, 0), and (2, 200, 0.01), are examined. The parameters $\{\theta_i\}$ in (1) are taken to be $(1/m)$.

3.1.2. Predictability of technical trading rules

Trend-following trading rules rest on the idea that a market trend in either direction, once established, has a strong tendency to persist for an extended period of time. Technical trading rules have advantages when little is known about the structure form of asset prices or returns. They utilize the price series itself and do not account for possible information available on the distribution of returns. Historically, moving averages are based upon asset prices. However, there are no clear reasons why they should be. Once some features of asset prices or returns distributions are known and the metric of the success of trading is well specified, better predictors can, in principle, be derived. If the

[2]Brock et al. (1992) also examined several other trading rules. The empirical analysis for those trading rules can be conducted similarly and the results support our general conclusions. For brevity, we choose not to report them in this paper.

market is predictable, no matter what the sources of such predictability are and which forms of the predictability are used, trading rules based on moving averages are inefficient in either a statistical or economic sense. As a demonstrative example, consider the following simple case.

Example 1. Assume that y_t follows the first-order autoregressive (AR(1)) process:

$$y_t = \phi y_{t-1} + \varepsilon_t, \quad (4)$$

where $\{\varepsilon_t\}$ are independently and identically normally distributed (i.i.d.) with zero mean and a unit variance. Then, for small $\phi > 0$, the one-period expected excess return from the (1, 3, 0) rule (without adjustment of interest rates and trading cost) is just about 90% of the maximum one-period expected excess return derived from the AR(1) model forecast (see the proof in Appendix A). Similar results should hold when x_t follows a non-linear process such as the AR-ARCH model.

3.1.3. Empirical results

In this section we evaluate performances of the 10 technical trading rules described in Section 3.1.1. We consider 'double-or-out' scheme in which a trader simply holds the Dow Jones portfolio in the days with no sell or buy signal, borrows at the risk-free interest rates (T-bill) to double the equity position in response to buy signals, and liquidates the portfolio in favor of T-bill when sell signals are given. This method was used to evaluate the predictive ability of technical trading rules by Brock et al. (1992). Bessembinder and Chan (1998) generalized the method to include risk free interest rates when they test the hypothesis that the market risk premium is negative, an interesting empirical finding from Brock et al. (1992). Due to the unavailability of the interest rate data over the full sample period, results with zero and an estimated monthly 0.3% interest rates are both reported. The robustness of the results to other levels of interest rates and for subperiods with and without non-synchronous trading is addressed in Section 4.3.

Table 2 displays the results for the 10 technical trading rules based on out-of-sample forecasts over the full sample period. The first column of each panel in Table 2 refers to different trading rules defined by (m_1, m_2, d). In panel 2A, N(buy) and N(sell) are the numbers of buy and sell days, respectively; N(trading) is the same as N_i in Appendix B, the number of days when new trading signals arrive to shift the position from 'double' to 'out' or vice versa; L(buy) and L(sell) are the average numbers of days staying in 'double' and 'out' positions, respectively. Numbers in the parentheses in the last two columns are the standard deviations of L(buy) and L(sell), respectively. It is apparent that there are more buy signals generated than sell signals for all trading rules across three Averages. As a consequence, L(buy) is longer than L(sell). Compared with the corresponding rule without the band, the number of transactions of the trading rule with a band reduces about 10% to 15%.

The columns labeled 'Buy' and 'Sell' in panel 2B present the quantities π_i^B/N(buy) and $-\pi_i^S/N$(sell), respectively, where π_i^B and π_i^S are as defined in Appendix B and N(buy) and N(sell) are as in panel 2A, with the difference between 'Buy' and 'Sell', denoted as 'Buy−Sell', listed aside. The t-ratios for the zero null hypotheses are in parentheses, with asterisks (double asterisks) indicating that the corresponding tests are statistically different from zero at the 5% (1%) level of significance. Buy signals generate higher average returns than sell signals for the Industrial and Transportation Averages, but not for the Utilities Average. This suggests that the dynamic properties of the Utilities Average differ from those of the In-

Table 2
Results for technical trading rules

Panel A: trading patterns					
(m_1, m_2, d)	N(buy)	N(sell)	N(trading)	L(buy)	L(sell)
Industrial					
(1, 50, 0)	15,320	10,611	1616	19.95 (30.42)	14.13 (19.89)
(1, 50, 0.01)	15,135	10,424	1428	22.55 (32.43)	16.02 (20.78)
(1, 150, 0)	16,512	10,119	816	40.59 (77.65)	25.80 (53.81)
(1, 150, 0.01)	16,436	10,017	708	46.83 (81.90)	29.69 (56.94)
(5, 150, 0)	16,708	10,317	422	78.49 (105.94)	49.90 (70.21)
(5, 150, 0.01)	16,601	10,171	386	85.85 (108.31)	54.51 (72.80)
(1, 200, 0)	16,961	9776	660	51.31 (114.26)	30.62 (68.19)
(1, 200, 0.01)	16,902	9697	568	59.65 (121.31)	35.55 (72.54)
(2, 200, 0)	17,065	9852	480	70.60 (134.52)	42.05 (77.70)
(2, 200, 0.01)	16,983	9754	434	78.04 (139.67)	46.54 (80.71)
Transportation					
(1, 50, 0)	14,646	11,191	1582	19.42 (29.11)	15.15 (21.80)
(1, 50, 0.01)	14,473	11,020	1364	22.57 (30.99)	17.52 (23.09)
(1, 150, 0)	15,551	11,032	736	43.04 (75.89)	30.98 (69.90)
(1, 150, 0.01)	15,502	10,965	648	48.93 (79.51)	35.15 (73.61)
(5, 150, 0)	15,737	11,202	380	83.42 (100.26)	59.96 (91.04)
(5, 150, 0.01)	15,653	11,098	336	94.38 (105.78)	67.77 (96.02)
(1, 200, 0)	15,824	10,801	644	49.20 (96.37)	34.54 (82.83)
(1, 200, 0.01)	15,745	10,737	566	55.94 (101.83)	39.34 (89.64)
(2, 200, 0)	15,896	10,881	492	64.38 (107.99)	45.23 (94.11)
(2, 200, 0.01)	15,815	10,809	434	72.88 (113.12)	51.29 (100.05)
Utilities					
(1, 50, 0)	9533	7481	890	22.40 (33.48)	17.81 (25.31)
(1, 50, 0.01)	9394	7365	764	26.10 (35.74)	20.75 (27.08)
(1, 150, 0)	10,114	7259	430	48.04 (88.92)	34.36 (64.42)
(1, 150, 0.01)	10,044	7198	362	57.03 (94.59)	40.81 (69.25)
(5, 150, 0)	10,207	7366	230	89.76 (109.6)	64.06 (88.93)
(5, 150, 0.01)	10,124	7276	216	95.61 (113.06)	68.14 (90.41)
(1, 200, 0)	9986	7377	390	52.21 (110.99)	38.14 (77.42)
(1, 200, 0.01)	9943	7317	328	62.07 (118.96)	45.67 (83.04)
(2, 200, 0)	10,035	7422	296	68.80 (123.61)	50.55 (86.25)
(2, 200, 0.01)	9980	7346	248	82.15 (131.34)	60.23 (91.62)

dustrial and Transportation Averages. The quantity 'Buy − Sell', which measures the predictive power for excess returns of the trading rule over the 'buy-and-hold' strategy (pre-trading costs), are significant at the 1% level for all three Averages. Furthermore, trading rules with bands generate higher returns than those without bands.

To evaluate the sensitivity of our results to interest rates, the break-even costs (see the definition of C_i in Appendix B) during the full sample period are computed with and without interest rates (panel 2C). When the interest rate is assumed to be zero ($r_t = 0$), the averages of the break-even costs (for all transactions including buys and sells) of the 10 trading rules are

Table 2. Continued

Panel B: daily buy and sell returns

(m_1, m_2, d)	Industrial			Transportation			Utilities		
	Buy	Sell	Buy − Sell	Buy	Sell	Buy − Sell	Buy	Sell	Buy − Sell
(1, 50, 0)	0.000462	−0.000161	0.000623	0.000538	−0.000447	0.000985	0.000504	−0.000537	0.001041
	(6.616)**	(−1.228)	(4.205)**	(6.570)**	(−3.450)**	(6.430)**	(5.733)**	(−3.426)**	(5.567)**
(1, 50, 0.01)	0.000479	−0.000166	0.000645	0.00055	−0.000468	0.001019	0.000528	−0.000555	0.001084
	(6.829)**	(−1.247)	(4.294)**	(6.669)**	(−3.572)**	(6.442)**	(5.967)**	(−3.514)**	(5.524)**
(1, 150, 0)	0.000391	−0.000173	0.000564	0.000456	−0.000356	0.000812	0.00037	−0.000442	0.000812
	(5.783)**	(−1.28)	(3.725)**	(5.707)**	(−2.741)**	(5.323)**	(4.306)**	(−2.739)**	(4.441)**
(1, 150, 0.01)	0.000402	−0.000185	0.000587	0.000459	−0.000377	0.000836	0.000363	−0.000445	0.000808
	(5.926)**	(−1.353)	(3.850)**	(5.736)**	(−2.890)**	(5.476)**	(4.198)**	(−2.738)**	(4.343)**
(5, 150, 0)	0.000364	−0.00013	0.00049	0.000439	−0.000271	0.00071	0.000263	−0.000321	0.000584
	(5.341)**	(−0.966)	(3.330)**	(5.492)**	(−2.138)*	(4.746)**	(3.045)**	(−2.024)*	(3.236)**
(5, 150, 0.01)	0.000375	−0.000144	0.00052	0.000444	−0.000271	0.000715	0.000258	−0.00033	0.000587
	(5.497)**	(−1.094)	(3.500)**	(5.528)**	(−2.119)*	(4.759)**	(2.971)**	(−2.053)*	(3.217)**
(1, 200, 0)	0.000385	−0.000205	0.000589	0.000417	−0.000283	0.0007	0.000278	−0.00037	0.000648
	(5.811)**	(−1.463)	(3.840)**	(5.245)**	(−2.171)*	(4.572)**	(3.275)**	(−2.313)*	(3.577)**
(1, 200, 0.01)	0.000396	−0.000215	0.000608	0.000428	−0.000298	0.000727	0.000274	−0.000378	0.000652
	(5.926)**	(−1.524)	(3.905)**	(5.372)**	(−2.277)*	(4.754)**	(3.220)**	(−2.345)*	(3.578)**
(2, 200, 0)	0.000387	−0.000158	0.000549	0.000409	−0.000278	0.000687	0.000267	−0.000337	0.000604
	(5.839)**	(−1.156)	(3.560)**	(5.153)**	(−2.148)*	(4.526)**	(3.130)**	(−2.113)*	(3.319)**
(2, 200, 0.01)	0.00039	−0.000177	0.000568	0.000413	−0.000283	0.000696	0.000275	−0.000351	0.000625
	(5.869)**	(−1.290)	(3.718)**	(5.183)**	(−2.179)*	(4.571)**	(3.202)**	(−2.182)*	(3.431)**
Average	0.000403	−0.000171	0.000574	0.000455	−0.000333	0.000789	0.000338	−0.000407	0.000745

Panel C: break-even costs (%)

	Industrial		Transportation		Utilities	
	$r_t = 0.0\%$	$r_t = 0.3\%$	$r_t = 0.0\%$	$r_t = 0.3\%$	$r_t = 0.0\%$	$r_t = 0.3\%$
(1, 50, 0)	0.272	0.249	0.407	0.391	0.495	0.478
(1, 50, 0.01)	0.314	0.289	0.481	0.462	0.592	0.572
(1, 150, 0)	0.503	0.444	0.748	0.702	0.806	0.757
(1, 150, 0.01)	0.597	0.529	0.868	0.816	0.943	0.884
(5, 150, 0)	0.874	0.760	1.309	1.219	1.093	1.001
(5, 150, 0.01)	0.997	0.872	1.481	1.379	1.154	1.055
(1, 200, 0)	0.646	0.564	0.750	0.691	0.704	0.654
(1, 200, 0.01)	0.769	0.674	0.879	0.812	0.834	0.774
(2, 200, 0)	0.850	0.737	0.968	0.891	0.872	0.806
(2, 200, 0.01)	0.962	0.838	1.105	1.018	1.068	0.988
Average	0.678	0.595	0.899	0.838	0.856	0.797

Trading rules are defined by the triple (m_1, m_2, d), where m_1 and m_2 are the lengths of the short and long averages, respectively, and d is the band. In panel A, N(buy) and N(sell) are the numbers of buy and sell days, N(trading) the number of trades, L(buy) the average number of days holding the Dow, L(sell) the average number of days being out of the market in favor of the T-bill, and numbers in the parentheses in the last two columns are standard deviations of L(buy) and L(sell), respectively. In panel B, numbers in parentheses are t-ratios for testing the zero null hypothesis, with asterisks (double asterisks) indicating that the corresponding tests are statistically different from zero at the 5% (1%) level of significance. In panel C, results are reported for the zero interest rate case and the case that the monthly interest rate equals 0.30%. Results are based on out-of-sample forecasts over the full sample period.

0.678%, 0.899% and 0.856% for the Industrial, Transportation and Utilities Averages, respectively. The results do not vary much when interest rates are taken into consideration. In fact, the bias due to the omission of interest rates is very limited because returns reported in the 'Buy − Sell' column are so much larger than the treasury-bill rates.[3] For example, if we use the 0.3% monthly rate, which is about the mean monthly T-bill return since January of 1926 to the later 1980s, the break-even cost of trading rule (1, 50, 0) for the Industrial Average is about 0.249%, a reduction of about 10% from its break-even cost 0.272% with $r_t = 0$.

3.2. Time series forecasts

3.2.1. Four autoregressive processes with GARCH in mean

Four autoregressive processes with GARCH in mean: AR(1), AR(1)-GARCH(1,1), AR(1)-GARCH(1,1)-M and AR(1)-EGARCH(1,1) are chosen to apply to the Dow data. The AR models with GARCH components make the variance of the residuals predictable and successfully capture the stylized facts of the conditional second moment of returns, such as thick tails and volatility clustering. The GARCH-M model extends the GARCH model to allow for the desired feature that the conditional variance can influence its conditional mean. Seasonal and non-synchronous effects can also be incorporated into either GARCH or GARCH-M models by making the intercept term time-dependent. The AR(1) portion would be less pronounced after the non-synchronous adjustment. Different from the GARCH and GARCH-M models which assume that the conditional volatility depends only upon the magnitude and not on the sign of unanticipated excess returns, the EGARCH model allows negative returns to predict higher volatility than positive returns of the same magnitude.

Table 3 reports the parameter estimates for these four time series models. Positive first-order autocorrelations are found in all three Averages. ARCH and EGARCH effects are always significant at the 1% level. However, the property that conditional variance can influence its conditional mean is confirmed in the Industrial and the Transportation Averages, but not in the Utilities Average.

Suppose that one trading period begins at time $t-1$. Based on the information set I_{t-1} available at time $t-1$, a simple trading strategy can be formulated as follows.

Definition 3. A buy (sell) signal is generated at time $t-1$ if

$$E(y_t|I_{t-1}) > \delta \ (< -\delta), \tag{5}$$

where I_τ is the information set at time τ and δ is a predetermined non-negative constant.

Note that I_t relies intimately on the model specification of y_t. The constant δ could be non-zero due to the consideration of some factors such as trading costs. In the absence of compelling reasons to choose a specific level, it is simply taken to be zero in our empirical analyses.

3.2.2. Predictability of the autoregressive processes with ARCH in mean

Having assumed autoregressive processes as embryos of asset returns, one may wonder how much of the gain or loss generated from these models can be expected. For some simple cases in the absence of transaction costs, we may derive approximate analytic results. In the case of AR(1)-GARCH(1,1), we find that total returns depend on both AR and GARCH parts.

[3] For example, the average monthly return of 10 technical trading rules for the Industrial Average is about 1.15%, based on the average daily return of 0.0574% (Table 2B), assuming 20 trading days per month.

Table 3
Parameter estimates for time series models

Panel A: AR(1)
$y_t = \mu + \phi y_{t-1} + \varepsilon_t$

	μ	ϕ
Industrial	0.000182	0.043310
	(2.67)**	(7.19)**
Transportation	0.000138	0.110440
	(1.76)	(18.42)**
Utilities	0.000051	0.082250
	(0.57)	(11.06)**

Panel B: AR(1)-GARCH(1,1)
$y_t = \mu + \phi y_{t-1} + \varepsilon_t$, $\varepsilon_t = \sqrt{h_t} e_t$, $h_t = w + \alpha \varepsilon_{t-1}^2 + \gamma h_{t-1}$, $e_t \sim IN(0, 1)$

	μ	ϕ	w	α	γ
Industrial	0.000425	0.094088	0.000001	0.096901	0.892479
	(8.416)**	(15.398)**	(25.697)**	(82.316)**	(593.863)**
Transportation	0.000380	0.124599	0.000002	0.097504	0.892925
	(7.026)**	(21.011)**	(20.359)**	(61.597)**	(484.004)**
Utilities	0.000276	0.182534	0.368×10^{-6}	0.098424	0.904185
	(5.780)**	(25.282)**	(18.320)**	(55.589)**	(528.395)**

Panel C: AR(1)-GARCH(1,1)-M
$y_t = \mu + \phi y_{t-1} + \delta \sqrt{h_t} + \varepsilon_t$, $\varepsilon_t = \sqrt{h_t} e_t$, $h_t = w + \alpha \varepsilon_{t-1}^2 + \gamma h_{t-1}$, $e_t \sim IN(0,1)$.

	μ	ϕ	w	α	γ	δ
Industrial	−0.000165	0.094665	0.000001	0.097650	0.891567	0.080400
	(−1.122)	(15.206)**	(25.206)**	(81.940)**	(586.868)**	(4.209)*
Transportation	0.000001	0.124554	0.000002	0.097553	0.892772	0.048714
	(0.004)	(20.903)**	(20.346)**	(61.563)**	(483.206)**	(2.355)*
Utilities	0.000181	0.182626	0.368×10^{-6}	0.098425	0.904161	0.018554
	(1.617)	(25.261)**	(18.363)**	(55.488)**	(528.217)**	(0.931)

Panel D: AR(1)-EGARCH(1,1)
$y_t = \mu + \phi y_{t-1} + \varepsilon_t$, $\varepsilon_t = \sqrt{h_t} e_t$, $\ln (h_t) = w + \alpha g(e_{t-1}) + \gamma \ln (h_{t-1})$, $g(e_t) = \beta e_t + (|e_t| - E|e_t|)$

	μ	ϕ	w	α	γ	β
Industrial	0.000169	0.100685	−0.201745	0.186006	0.977808	−0.413082
	(2.952)**	(15.185)**	(−15.314)**	(30.521)**	(696.268)**	(−20.841)**
Transportation	0.000169	0.128061	−0.152729	0.189867	0.982642	−0.290091
	(2.490)**	(18.655)**	(−12.161)**	(27.333)**	(724.346)**	(−16.777)**
Utilities	0.000149	0.178892	−0.062382	0.185201	0.992608	−0.232449
	(2.911)**	(23.487)**	(−7.221)**	(22.566)**	(1133.124)**	(−11.521)**

Results are based upon the sample period from May 24, 1896 to May 24, 1996 for the Industrial Average, from October 26, 1896 to May 24, 1996 for the Transportation Average, and from January 2, 1929 to May 24, 1996 for the Utilities Average. The AR(1) is estimated by OLS. The AR(1)-GARCH, AR(1)-GARCH-M and AR(1)-EGARCH are estimated using maximum likelihood. The numbers in parentheses are *t*-ratios. The *t*-ratios marked with asterisks (double asterisks) indicate that the corresponding coefficients are statistically different from zero at the 5% (1%) level of significance.

The result is very sensitive to the level of the ARCH coefficient.

Example 2. Suppose the return y_t follows AR(1)-GARCH(1,1) process defined as:

$$y_t = \phi y_{t-1} + \sigma_t e_t, \tag{6}$$

$$\sigma_t^2 = w + \alpha \sigma_{t-1}^2 e_{t-1}^2 + \gamma \sigma_{t-1}^2, \tag{7}$$

where $w > 0$, α, $\gamma \geq 0$, $\alpha + \gamma < 1$, and $\{e_t\}$ are i.i.d. normal with mean zero and variance 1. Then, for small α and γ, the maximum one-period expected excess return is approximately equal to

$$\frac{\sqrt{2w}|\phi|}{[\pi(1-\phi^2)(1-\alpha-\gamma)]^{1/2}}. \tag{8}$$

The proof is given in Appendix A.

3.2.3. Empirical results

Table 4 displays the results of the four autoregressive models with ARCH in mean described in Section 3.2.1, denoted by AR, AR-GARCH, AR-GARCH-M, and AR-EGARCH, respectively. They are reported here only to demonstrate some properties in comparison with those of technical trading rules in Table 2 and to motivate the approach of the combined strategies discussed in Section 4. The structures of the three panels are similar to those in Table 2. For illustrative purposes, we will report only the results based on models with parameters estimated over the full sample period (Table 3).

From panel 4A, N(trading)s are much greater than those for technical trading rules, and L(buy)s and L(sell)s range only from 1.4 to 4.8. In contrast to the results of the technical trading rules reported in Table 2, the sell signals generate higher returns than buy signals for all models across the three Averages during the time period considered (panel 4B). With only one exception, i.e. the case of the AR-EGARCH model applied to the Industrial Average, the 'Buy' and 'Sell' are very close. This finding suggests that technical trading rules and time series forecasts capture different aspects of market predictability, and hence, strategies combining the two methods might yield more favorable results. As in the cases of technical trading rules, evidence against the hypotheses that 'Buy − Sell' returns are zero is strong for all four models. Because of the larger number of transactions required (panel 4A),[4] the break-even costs reported in panel 4C are much lower than those for technical trading rules (panel 2C).

4. Combined trading strategies

4.1. Definition

Trading strategies combining technical trading rules and time series forecasts can be defined as follows.

Definition 4. A buy (sell) signal is generated at time $t-1$ if both the technical trading rule and the time series model emit buy (sell) signal based on I_{t-1}.

This combined trading strategy employs different aspects of the predictive power from technical trading rules and from time series forecasts. Its special design requires a remarkably lower number of transactions. Although possibly not sensitive enough to generate early signals, it is able to capture 'big moves' over the long run and has the high potential to yield excess returns in the costly trading environment.

Since the AR(1) component appears to be most important in predicting returns (Sections 2 and 3.2.3), the AR(1) model is used as our parsimonious time series model in the empirical

[4] One reason for this larger number of transactions is that δ in (5) is chosen to be 0, hence a buy (sell) is indicated whenever the conditional expected return is positive (negative).

Table 4
Results for time series models

Panel A: trading patterns

	N(buy)	N(sell)	N(trading)	L(buy)	L(sell)
Industrial					
AR	20,366	7229	4882	4.17 (4.21)	1.48 (0.84)
AR-GARCH	20,624	6981	4750	4.34 (4.54)	1.47 (22.46)
AR-GARCH-M	21,480	6115	4434	4.84 (4.35)	1.38 (0.71)
AR-EGARCH	17,127	10,468	6054	2.83 (2.25)	1.73 (1.10)
Transportation					
AR	15,414	12,006	3277	4.70 (9.09)	3.66 (6.07)
AR-GARCH	18,499	8968	5382	3.44 (3.11)	1.67 (1.07)
AR-GARCH-M	18,845	8622	5340	3.53 (3.05)	1.61 (1.00)
AR-EGARCH	15,992	11,472	6072	2.63 (2.08)	1.89 (1.29)
Utilities					
AR	10,075	7877	3956	2.55 (2.15)	1.99 (1.46)
AR-GARCH	11,206	6746	3780	3.02 (2.81)	1.82 (1.25)
AR-GARCH-M	11,348	6604	3729	3.04 (2.79)	1.77 (1.19)
AR-EGARCH	10,321	7631	3908	2.64 (2.28)	1.95 (1.41)

Panel B: daily buy and sell returns

	Industrial			Transportation			Utilities		
	Buy	Sell	Buy − Sell	Buy	Sell	Buy − Sell	Buy	Sell	Buy − Sell
AR	0.000492	−0.000700	0.001192	0.001044	−0.001101	0.002145	0.000904	−0.001039	0.001943
	(6.190)**	(−5.779)**	(8.227)**	(12.200)**	(−9.434)**	(14.823)**	(9.035)**	(−7.587)**	(11.456)**
AR-GARCH	0.000483	−0.000716	0.001198	0.000856	−0.001343	0.002199	0.000789	−0.001174	0.001963
	(7.244)**	(−4.274)**	(6.650)**	(11.013)**	(−9.494)**	(13.624)**	(8.518)**	(−7.541)**	(10.836)**
AR-GARCH-M	0.000449	−0.000768	0.001217	0.000828	−0.001370	0.002199	0.000753	−0.001154	0.001914
	(6.425)**	(−3.257)**	(4.950)**	(10.232)**	(−10.163)**	(13.980)**	(8.011)**	(−7.459)**	(10.534)**
AR-EGARCH	0.000676	−0.000633	0.001308	0.001036	−0.001114	0.002150	0.000891	−0.001084	0.001975
	(9.145)**	(−5.158)**	(9.138)**	(12.176)**	(−9.461)**	(14.801)**	(9.055)**	(−7.717)**	(11.515)**
Average	0.000525	−0.000704	0.001229	0.000941	−0.001232	0.002173	0.000834	−0.001113	0.001949

Panel C: break-even costs (%)

	Industrial		Transportation		Utilities	
	$r_t = 0.0\%$	$r_t = 0.3\%$	$r_t = 0.0\%$	$r_t = 0.3\%$	$r_t = 0.0\%$	$r_t = 0.3\%$
AR	0.154	0.141	0.447	0.442	0.219	0.216
AR-GARCH	0.157	0.143	0.259	0.250	0.222	0.216
AR-GARCH-M	0.162	0.144	0.257	0.243	0.217	0.210
AR-EGARCH	0.150	0.145	0.242	0.238	0.223	0.220
Average	0.156	0.143	0.301	0.294	0.220	0.216

In panel A, N(buy) and N(sell) are the numbers of buy and sell days, N(trading) the number of trades, L(buy) the average number of days holding the Dow, L(sell) the average number of days being out of the market in favor of the T-bill, and numbers in the parentheses in the last two columns are standard deviations of L(buy) and L(sell), respectively. In panel B, numbers in parentheses are t-ratios for testing the zero null hypothesis, with asterisks (double asterisks) indicating that the corresponding tests are statistically different from zero at the 5% (1%) level of significance. In panel C, results are reported for the zero interest rate case and the case that the monthly interest rate equals 0.30%. Model parameters are estimated over the full sample period and are given in Table 3.

analysis. In order to evaluate the performance of the AR(1) model with parameters estimated based on the available data, we used the following rolling technique: at day *t*, estimated model parameters are based on a window consisting of a fixed number of historical data available at day *t*.[5] To reduce the risk of the data snooping bias, the window lengths are taken to be 50, 150, and 200 days, the same lengths of the long moving averages used in the technical trading rules.[6]

4.2. Empirical results

The 10 combined trading strategies evaluated are based on the 10 technical trading rules examined in Section 3.2 and the rolling AR(1) model with the rolling window size equal to the length of the long moving average. Following Definition 4, a buy (sell) signal is generated if both the rolling AR(1) model and the corresponding technical trading rule emit buy (sell) signal. The empirical results based on out-of-sample forecasts over the full sample period are reported in Table 5.

[5]Use of rolling windows is, of course, not new. For example, Fama and Macbeth (1973) applied a 'rolling regression' strategy to estimate conditional betas at day *t* using the return data for a period of 5 to 8 years prior to the day *t*, and Foster and Nelson (1996) studied rolling sample variance estimators. The main reason for using only the most recent part of the historical data, rather than all available data, is to allow the model parameters to change over time.

[6]From a conceptual point of view, since we are not focusing on the predictability in the second (or higher) moments of returns, the otherwise important difference between pure AR(1) and AR(1) with GARCH types of disturbances becomes irrelevant (Fama, 1970). It is important to note, however, that the inclusion of GARCH components in the model is empirically relevant (although its impact on the final results is negligible for our data) since the estimates of AR(1) and the evaluation of returns depend on both AR and GARCH parts (see Example 2 in Section 3.2.2).

The first columns in the three panels of Table 5 list the 10 combined trading strategies labeled by (m_1, m_2, d, M), where (m_1, m_2, d) refers to the corresponding technical trading rule and M indicates the window size of the rolling AR(1) model (which is always equal to m_2 in our specification). From panel 5A, the required number of transactions is reduced significantly, in comparison with those from corresponding technical trading rules (panel 2A). Averages of buy returns are always less than sell returns for all three Averages (panel 5B). The entries in the 'Buy − Sell' column indicate that the excess returns (pre-trading cost) are significantly different from zero. From the comparison of panel 5C with panel 2C, the break-even costs for combined strategies are always greater than those for the corresponding technical trading rules and could be as much as one and one-half times greater. The average break-even costs without accounting for interest rates for the Industrial, Transportation and Utilities Averages are 1.130%, 2.044% and 1.841%; while assuming monthly rate being 0.30%, they become 1.009%, 1.964% and 1.764%. In some cases, they could be as much as 120% higher than those of the corresponding technical trading rules (panel 2C).

4.3. Robustness analysis

To test the robustness of our results, we repeat the above analysis for four subperiods, with and without non-synchronous trading adjustment, and for various different levels of monthly interest rates. While only the results for the Industrial Average are reported (Table 6), our conclusions also apply, in principle, to the Transportation and Utilities Averages.

The subperiods are divided equally to allow for the same number of observations: 1896–1920, 1921–1945, 1946–1970, and 1971–1996. The entries in panels 6A1–A2 and panels 6B1–B2 are the averages for the results from the 10 technical trading rules and from the 10 combined trading strategies with and without the non-synchronous trading adjustment, respectively. Overall, the combined trading strategies outperform the corresponding technical trading rules, especially for the second half of the first 100 years of the Dow history. Also, the break-even costs of the combined trading strategies are more than twice those of the technical trading rules during the third and the fourth subperiods. These results retain regardless of the non-synchronous trading adjustment.

The conclusion holds, at least qualitatively, when other reasonable levels of interest rates are considered. When monthly rate is assumed to be 0.6% (which is twice the historical

Table 5
Results for strategies combining technical trading rules and time series forecasts

Panel A: trading patterns (m_1, m_2, d, M)	N(buy)	N(sell)	N(trading)	L(buy)	L(sell)
Industrial					
(1, 50, 0, 50)	12,593	7195	1140	28.62 (40.44)	19.69 (24.43)
(1, 50, 0.01, 50)	12,481	7109	1046	31.2 (41.88)	21.46 (25.28)
(1, 150, 0, 150)	13,434	6051	636	52.91 (88.60)	32.27 (65.63)
(1, 150, 0.01, 150)	13,368	6019	580	57.89 (91.28)	35.51 (67.98)
(5, 150, 0, 150)	13,389	6007	326	103.12 (124.08)	63.07 (85.42)
(5, 150, 0.01, 150)	13,323	5944	314	107.03 (124.96)	65.51 (86.32)
(1, 200, 0, 200)	13,650	5749	496	68.14 (135.02)	40.89 (79.40)
(1, 200, 0.01, 200)	13,616	5723	442	76.45 (140.89)	45.89 (83.00)
(2, 200, 0, 200)	13,652	5737	342	98.65 (164.28)	59.44 (89.35)
(2, 200, 0.01, 200)	13,605	5696	320	105.46 (167.86)	63.51 (91.15)
Transportation					
(1, 50, 0, 50)	11,371	7928	1176	26.24 (35.51)	20.26 (25.77)
(1, 50, 0.01, 50)	11,275	7838	1072	28.83 (36.94)	22.18 (26.68)
(1, 150, 0, 150)	11,829	7192	572	56.06 (86.93)	39.18 (79.94)
(1, 150, 0.01, 150)	11,801	7171	518	61.77 (89.65)	43.40 (83.02)
(5, 150, 0, 150)	11,761	7116	284	113.41 (120.93)	78.44 (101.92)
(5, 150, 0.01, 150)	11,713	7081	260	123.72 (122.15)	85.84 (104.91)
(1, 200, 0, 200)	11,846	6990	510	62.48 (110.25)	43.25 (94.93)
(1, 200, 0.01, 200)	11,804	6959	462	68.93 (114.69)	47.79 (100.27)
(2, 200, 0, 200)	11,823	6971	344	92.68 (129.14)	64.09 (112.21)
(2, 200, 0.01, 200)	11,777	6939	310	102.7 (133.95)	71.17 (116.38)
Utilities					
(1, 50, 0, 50)	7477	5357	682	29.42 (39.32)	23.07 (30.41)
(1, 50, 0.01, 50)	7402	5318	600	33.41 (40.93)	26.26 (32.56)
(1, 150, 0, 150)	7582	4621	354	58.79 (99.33)	41.26 (69.63)
(1, 150, 0.01, 150)	7545	4606	302	68.81 (104.69)	48.42 (76.50)
(5, 150, 0, 150)	7531	4577	192	108.33 (125.39)	75.98 (94.66)
(5, 150, 0.01, 150)	7496	4542	180	115.62 (127.75)	80.92 (96.88)
(1, 200, 0, 200)	7340	4526	304	65.20 (125.13)	49.38 (87.30)
(1, 200, 0.01, 200)	7327	4513	260	76.19 (132.84)	57.78 (92.37)
(2, 200, 0, 200)	7331	4515	234	84.96 (141.75)	63.90 (95.61)
(2, 200, 0.01, 200)	7306	4494	200	99.36 (166.17)	74.81 (100.24)

Table 5. Continued

Panel B: daily buy and sell returns

(m_1, m_2, d, M)	Industrial			Transportation			Utilities		
	Buy	Sell	Buy − Sell	Buy	Sell	Buy − Sell	Buy	Sell	Buy − Sell
(1, 50, 0, 50)	0.000533	−0.000329	0.000861	0.000812	−0.000879	0.001691	0.000773	−0.000835	0.001608
	(7.033)**	(−1.952)	(4.661)**	(8.857)**	(−5.446)**	(9.106)**	(7.835)**	(−4.192)**	(7.198)**
(1, 50, 0.01, 50)	0.000543	−0.000334	0.000878	0.000817	−0.000899	0.001716	0.000784	−0.000858	0.001641
	(7.141)**	(−1.963)*	(4.074)**	(8.853)**	(−5.518)**	(9.164)**	(7.902)**	(−4.306)**	(7.369)**
(1, 150, 0, 150)	0.000497	−0.000699	0.001197	0.000856	−0.001165	0.002021	0.000655	−0.001235	0.001891
	(6.826)**	(−3.715)**	(5.762)**	(9.917)**	(−6.853)**	(10.593)**	(7.191)**	(−5.663)**	(7.993)**
(1, 150, 0.01, 150)	0.000503	−0.000711	0.001214	0.000856	−0.001171	0.002028	0.000636	−0.001244	0.00188
	(6.884)**	(−3.764)**	(5.972)**	(9.908)**	(−6.875)**	(10.626)**	(6.963)**	(−5.685)**	(7.928)**
(5, 150, 0, 150)	0.000476	−0.00067	0.001146	0.000826	−0.001124	0.00195	0.000576	−0.00121	0.001785
	(6.454)**	(−3.636)**	(5.744)**	(9.489)**	(−6.652)**	(10.244)**	(6.208)**	(−5.505)**	(7.489)**
(5, 150, 0.01, 150)	0.00048	−0.00068	0.001161	0.000832	−0.001135	0.001967	0.000566	−0.001209	0.001775
	(6.500)**	(−3.666)**	(5.805)**	(9.523)**	(−6.688)**	(10.322)**	(6.091)**	(−5.463)**	(7.401)**
(1, 200, 0, 200)	0.000499	−0.000636	0.001136	0.000859	−0.001062	0.001921	0.000663	−0.001188	0.001851
	(7.042)**	(−3.208)**	(5.405)**	(9.893)**	(−6.219)**	(10.026)**	(7.754)**	(−5.247)**	(7.644)**
(1, 200, 0.01, 200)	0.000503	−0.000648	0.001151	0.000866	−0.001073	0.001939	0.000669	−0.001196	0.001866
	(7.073)**	(−3.255)**	(5.460)**	(9.954)**	(−6.259)**	(10.092)**	(7.815)**	(−5.269)**	(7.691)**
(2, 200, 0, 200)	0.000503	−0.000609	0.001111	0.000852	−0.001097	0.001949	0.000643	−0.001162	0.001805
	(7.052)**	(−3.151)**	(5.407)**	(9.818)**	(−6.448)**	(10.214)**	(7.485)**	(−5.116)**	(7.409)**
(2, 200, 0.01, 200)	0.000508	−0.000627	0.001134	0.000854	−0.00109	0.001944	0.000653	−0.001172	0.001826
	(7.125)**	(−3.223)**	(5.482)**	(9.817)**	(−6.383)**	(10.946)**	(7.600)**	(−5.137)**	(7.483)**
Average	0.000505	−0.000594	0.00110	0.000843	−0.001070	0.001912	0.000662	−0.001131	0.001793

Panel C: break-even costs (%)

(m_1, m_2, d, M)	Industrial		Transportation		Utilities	
	$r_t = 0.0\%$	$r_t = 0.3\%$	$r_t = 0.0\%$	$r_t = 0.3\%$	$r_t = 0.0\%$	$r_t = 0.3\%$
(1, 50, 0, 50)	0.398	0.362	0.689	0.667	0.752	0.728
(1, 50, 0.01, 50)	0.438	0.399	0.758	0.734	0.863	0.837
(1, 150, 0, 150)	0.860	0.771	1.617	1.557	1.504	1.441
(1, 150, 0.01, 150)	0.949	0.854	1.786	1.719	1.738	1.665
(5, 150, 0, 150)	1.595	1.425	3.119	2.996	2.558	2.443
(5, 150, 0.01, 150)	1.664	1.488	3.419	3.285	2.69	2.568
(1, 200, 0, 200)	1.056	0.937	1.725	1.654	1.685	1.615
(1, 200, 0.01, 200)	1.194	1.060	1.915	1.836	1.982	1.900
(2, 200, 0, 200)	1.513	1.339	2.575	2.47	2.128	2.038
(2, 200, 0.01, 200)	1.638	1.452	2.841	2.724	2.51	2.405
Average	1.130	1.009	2.044	1.964	1.841	1.764

Trading rules are defined by (m_1, m_2, d, M), where m_1, m_2 and d are parameters specifying the technical trading rules, and M is the window size for the rolling AR(1) model. In panel A: N(buy) and N(sell) are the numbers of buy and sell days; N(trading) the number of trades; L(buy) the average number of days holding the Average; L(sell) the average number of days being out of the market in favor of the T-bill; and numbers in the parentheses in the last two columns are standard deviations of L(buy) and L(sell), respectively. In panel B, numbers in parentheses are t-ratios for testing the zero null hypothesis, with asterisks (double asterisks) indicating that the corresponding tests are statistically different from zero at the 5% (1%) level of significance. In panel C, results are reported for the zero interest rate case and the case that the monthly interest rate equals 0.30%. Results are based on out-of-sample forecasts over the full sample period.

Table 6
Results of robustness analysis

Subperiod	Buy return	Sell return	Buy − Sell return	Cost (%) ($r_t = 0$/month)	Cost (%) ($r_t = 0.3$/month)	Cost (%) ($r_t = 0.6$/month)
Panel A1: technical trading rules with non-synchronous trading adjustment						
1896–1920	0.000391	−0.000290	0.000681	0.642	0.608	0.597
1921–1945	0.000445	−0.000411	0.000856	1.041	0.941	0.908
1946–1970	0.000386	−0.000044	0.000430	0.633	0.523	0.486
1971–1996	0.000399	−0.000191	0.000590	0.477	0.366	0.329
Panel B1: combined trading strategies with non-synchronous trading adjustment						
1896–1920	0.000327	−0.000308	0.000635	0.609	0.539	0.516
1921–1945	0.000517	−0.000576	0.001090	1.421	1.247	1.189
1946–1970	0.000638	−0.001197	0.001830	1.471	1.344	1.302
1971–1996	0.000507	−0.000459	0.000967	1.143	1.001	0.954
Panel A2: technical trading rules without non-synchronous trading adjustment						
1896–1920	0.000397	−0.000291	0.000688	0.661	0.627	0.616
1921–1945	0.000473	−0.000421	0.000894	1.103	1.003	0.970
1946–1970	0.000414	−0.000062	0.000475	0.642	0.538	0.504
1971–1996	0.000428	−0.000071	0.000499	0.603	0.493	0.456
Panel B2: combined trading strategies without non-synchronous trading adjustment						
1896–1920	0.000340	−0.000304	0.000644	0.635	0.566	0.543
1921–1945	0.000546	−0.000559	0.001100	1.472	1.298	1.241
1946–1970	0.000657	−0.001173	0.001831	1.518	1.391	1.349
1971–1996	0.000535	−0.000596	0.001132	1.292	1.150	1.102

Results are presented for four subperiods and for the Industrial Average only. The daily return is measured as log differences of the Average. Ten technical trading rules used alone and 10 corresponding combined trading strategies are evaluated.

estimate),[7] the average break-even costs (with the non-synchronous trading adjustment) are only slightly less than those with the 0.3% monthly rate.

5. Conclusions

In this study, two different but closely related measurements for the performance of various trading strategies are evaluated. The first measurement, 'Buy − Sell', assesses the excess gain in returns over the buy-and-hold (pre-trading cost). Utilizing the first 100-year Dow Jones Averages, we find that both technical trading rules and time series models have forecasting ability but each of them captures only part of the aspects of market predictability. Our empirical results indicate that when the market rises the technical rules perform typically better than strategies derived from the conventional time series models. However, the time series forecasts are in general superior to the technical rules when the market falls. Motivated by this finding, we develop a simple approach which combines technical rules and time series fore-

[7] As many previous studies did (Allen & Karjalainen, 1999; Bessembinder & Chan, 1998), we assume that one can borrow at the risk-free rates corresponding to Treasury Bills. Since the Treasury doesn't engage in margin transactions, a referee suggested that the call margin rates would be an appropriate measure. If this is the case, the borrowing rate is likely to be higher (than 0.3% per month), especially in the earlier sample period.

casts. Our empirical results reveal that, compared with the technical trading rules, the average 'Buy − Sell' of combined trading strategies (with the non-synchronous trading adjustment) are increased by about 92%, 142% and 141% for the Industrial, Transportation and Utilities Averages, respectively.

The second measurement, 'break-even cost', values the average cost that absorbs the excess returns derived from the trading strategy over the buy-and-hold. It depends not only upon the excess return measurement, but also on the number of transactions required. Because of its design, the number of trades of the combined trading strategy is considerably less than that required by the corresponding technical trading rule. If the monthly interest rate of 0.30% is assumed over the full sample period, the average break-even costs from the 10 combined trading strategies (with the non-synchronous trading adjustment) are about 1.009%, 1.964%, and 1.764% for the Industrial, Transportation and Utilities Averages, respectively, in comparison to about 0.595%, 0.838% and 0.797% from the 10 corresponding technical trading rules considered. While larger than some estimates of actual trading costs on the NYSE (about 0.25% plus market impact (Chan & Lakonishok, 1993) or about 25 cents per share including both brokerage costs and an allowance for the bid-ask spread (Fluck, Malkiel, & Quandt, 1997)), the break-even costs from the technical trading rules are within the range of other estimates, for example, 1.35% by Stoll and Whaley (1983).[8]

Because of the growing evidence on the predictability on asset returns documented in the recent literature, an approach to utilize the information on the predictability looks promising and may have some practical value. The combination of commonly used technical trading rules and some popular time series forecasts, such as the combined trading strategies discussed in this study, is one attempt in this direction. Although there are various seemly plausible models to capture the predictability in asset returns, the question of which time series models should be used and how to formulate a trading strategy (combined with some existing trading techniques such as technical trading rules) must be resolved with the particular application and data at hand. It is apparent that our simple combined trading strategy can be improved depending on different time horizons of investment and the level of transaction costs. Other market data such as trading volume may also improve the excess returns. Since our objective in this paper is to provide further empirical evidence on predictability of asset returns, we keep our discussion on some simple and intuitive approaches to reduce the risk of data snooping problems (Bossaerts & Hillion, 1999).

While not necessarily implying inefficiency in the US stock market, our results do, however, have practical implications on the formulation of economic models of asset prices and investment strategies. The evidence of the existence of market predictability suggest that some forms of asset returns, such as that used in the standard Black–Scholes American option formula, are unlikely to be plausible models. Whether the predictability of returns and opportunities for excess returns represent evidence of time-varying risk premiums, the irrational 'animal spirits' of agents, market inefficiency or other complex implications remains an open question. The form of the market predictability is poorly understood and finding the source of excess returns is a difficult task. We are currently investigating the nature of the switching point of the combined trading strategies. The results should be of interest to further explore the interaction between technical analysis and time series forecasts and to provide some sort of

[8] We note, however, that in order to obtain excess returns the 'break-even costs' may vary for different time periods because transaction costs change substantially from 1896 to 1996.

synthesis about the underlying behavior of stock price changes.

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Appendix A. Proofs of Examples 1 and 2

For simplicity, we take the risk free interest and the trading cost rate as being 0. Then, it is trivial that the strategy at time $t-1$ which maximizes the expected one-period excess return is described in Definition 3 with $\delta = 0$. From (6), $y_t = \sigma_t e_t + \phi \sum_{j=1}^{\infty} \phi^{j-1} \sigma_{t-j} e_{t-j}$. It is well known that $E(\sigma_t e_t) = 0$, $E(\sigma_t^2 e_t^2) = w/(1-\alpha-\gamma)$, and $\{\sigma_t e_t\}$ is a series of martingale difference. Hence,

$$E\left[\phi \sum_{j=1}^{\infty} \phi^{j-1} \sigma_{t-j} e_{t-j}\right] = 0$$

and

$$E\left\{\left[\phi \sum_{j=1}^{\infty} \phi^{j-1} \sigma_{t-j} e_{t-j}\right]^2\right\} = w\phi^2/[(1-\phi^2)(1-\alpha-\gamma)].$$

Since $E(y_t | I_{t-1}) = \phi \sum_{j=1}^{\infty} \phi^{j-1} \sigma_{t-j} e_{t-j}$, the maximum one-period expected excess return is

$$G_{\mathrm{M}} = E[y_t 1_{\{E(y_t|I_{t-1})>0\}} - y_t 1_{\{E(y_t|I_{t-1})<0\}}] = 2E\left\{\phi \sum_{j=1}^{\infty} \phi^{j-1} \sigma_{t-j} e_{t-j} 1_{\{\phi \sum_{j=1}^{\infty} \phi^{j-1} \sigma_{t-j} e_{t-j} > 0\}}\right\}.$$

Assume that when α and γ are small in absolute value so that we can approximately treat $\sum_{j=1}^{\infty} \phi^{j-1} \sigma_{t-j} e_{t-j}$ in the right hand side of the above as normal, then

$$G_{\mathrm{M}} = \frac{\sqrt{2w}|\phi|}{[\pi(1-\phi^2)(1-\alpha-\gamma)]^{1/2}},$$

which is (8). For the model (4) with small $\phi > 0$, (8) becomes

$$G_{\mathrm{M}} = \frac{\sqrt{2}\phi}{[\pi(1-\phi^2)]^{1/2}} = \frac{\sqrt{2}\phi}{\pi^{1/2}}.$$

Let $G_{(1,3,0)}$ be the expected one-period ahead excess return of the strategy based on the (1, 3, 0) rule. For this rule,

$$B = \{x_{t-1} > (x_{t-2} + x_{t-3})/2\} \quad \text{and}$$
$$S = \{x_{t-1} < (x_{t-2} + x_{t-3})/2\}.$$

Thus,

$$\begin{aligned} G_{(1,3,0)} &= 2E[y_t 1_{\{x_{t-1} > (x_{t-2}+x_{t-3})/2\}}] \\ &= 2\phi E[y_{t-1} 1_{\{e^{y_{t-1}} > (1+e^{-y_{t-2}})/2\}}]. \end{aligned}$$

Since the distribution of y_{t-2} is normal with mean zero and variance equal to $1/(1-\phi^2)$, and $y_{t-1} = \phi y_{t-2} + \varepsilon_{t-2}$, where ε_{t-2} is independent of y_{t-2} and is normally distributed with mean zero and unit variance,

$$\begin{aligned} &[y_{t-1} 1_{\{e^{y_{t-1}} > (1+e^{-y_{t-2}})/2\}}] \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\{-v^2/2 - \log^2[(1+e^{-v})/2]/2\}\, dv \\ &= 0.893 \frac{1}{\sqrt{2\pi}} \end{aligned}$$

when ϕ is small.

Appendix B. Calculations of break-even trading costs

Since the 'double-or-out' strategy is fully described by the previous studies, we only provide a brief summary.

Let $\pi_{i,t}$ denote the excess return earned by applying trading rule i (prior to trading cost) on day t, then

$$\pi_{i,t} = \begin{cases} y_t - r_t, & \text{if trading rule } i \text{ yields buy signal at day } t-1 \\ 0, & \text{if no trading signal at day } t-1 \\ -(y_t - r_t), & \text{if trading rule } i \text{ yields sell signal at day } t-1 \end{cases}$$

where r_t is the interest rate on day t. The total excess returns over the sample period due to the trading strategy is then

$$\pi_i \equiv \pi_i^B + \pi_i^S,$$

where

$$\pi_i^B \equiv \sum_{\text{buy days}} \pi_{i,t} \quad \text{and} \quad \pi_i^S \equiv \sum_{\text{sell days}} \pi_{i,t}.$$

To examine whether trading strategy i could be profitably used in a costly trading environment, we follow Bessembinder and Chan (1998) to compute its 'break-even costs' C_i, which is defined as

$$C_i = \frac{\pi_i}{2N_i},$$

where N_i is the number of days when a new trading signal arrives to shift the position from 'double' to 'out' or vice versa. The factor 2 in the denominator of C_i is due to the design of the 'double-or-out' strategy.

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