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# Statistical physics in foreign exchange currency and stock markets ☆

M. Ausloos

*B5, S.U.P.R.A.S., and G.R.A.S.P. University of Liège, B-4000 Liège, Belgium*

**Abstract**

Problems in economy and finance have attracted the interest of statistical physicists all over the world. Fundamental problems pertain to the existence or not of long-, medium- or/and short-range power-law correlations in various economic systems, to the presence of financial cycles and on economic considerations, including economic policy. A method like the detrended fluctuation analysis is recalled emphasizing its value in sorting out correlation ranges, thereby leading to predictability at short horizon. The $(m,k)$-Zipf method is presented for sorting out short-range correlations in the sign and amplitude of the fluctuations. A well-known financial analysis technique, the so-called moving average, is shown to raise questions to physicists about fractional Brownian motion properties. Among spectacular results, the possibility of crash predictions has been demonstrated through the log-periodicity of financial index oscillations. © 2000 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

In this paper, as I did in the pertinent oral contribution, I intend to cover in a rather practical way, or within a useful framework, numerical and theoretical investigations

☆ Performance results listed in the paper and in all marketing materials represent simulated computer results over past historical data, and not the results of an actual account. Hypothetical or simulated performance results have certain limitations. Unlike any actual performance record, simulated results do not represent actual trading. Also, since the trades have not actually been executed, the results may have under-or-over compensated for the impact, if any, of certain market factors, such as lack of liquidity. Simulated programs in general are also subject to the fact that they are designed with the benefit of hindsight. No representation is being made that any account will or is likely to achieve profits or losses similar to those shown. Testimonial or actual account results presented do not necessarily reflect the results of all users of the program. Past performance does not guarantee future results.

*E-mail address:* ausloos@gw.unipc.ulg.ac.be (M. Ausloos).


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on financial applications of statistical physics as they have been covered up to this date by the Liège group. Emphasis will be on outlining findings reported in previous publications, and their related technique. No attempt is made to provide an extensive review of the literature. In the spirit of the lectures as requested by the organizers, transparence and pedagogy should hopefully be the main pillars of what follows. Moreover, not all practical techniques will be covered. More specifically, the following contribution is divided in four sections: (1) the detrended fluctuation analysis method, (2) the Zipf technique, (3) the moving-average (analysis) technique, (4) the log-periodic time-series analysis near crashes. As in the oral lecture, the multifractal analysis technique is mentioned only when and if useful for the approaches and topics to which it can be connected. The wavelet method [1] and the quasi-equilibrium thermodynamics approach [2] are not touched upon here. Many interesting topics, basic and advanced ones can be found in the recent book by Mantegna and Stanley [3].

As a preliminary statement let it be recalled that even though the error bar sizes are not often mentioned, the author has much practice in critical exponent analysis and their extraction from experimental data. He has shown persistency [4–9] in getting the best technique available for sorting out numerical values. He is convinced that he cared as much about the results from nonlinear fits of financial time series, and subsequent data, as he did in previous work on critical phase transitions, as e.g. in Refs. [4–9]. Moreover even though not all analyzed data technique and results have been checked with respect to standard physical signals (like fractional Brownian motion, white noise, etc.) most results when published were thought to be sufficiently in agreement with what should be thereby expected. Nevertheless, it is necessary to warn against mistreatment of data, and as pointed out elsewhere [10], one can sometimes obtain quite varied mean values, large error bars or even unreliable parameters. It is however extremely difficult to give general rules about how to be satisfied with extracted data/parameter values from nonlinear fit techniques. More sophisticated statistical techniques than I have used or I am familiar with can surely be used with modern computers. However, there is no strict need to request extreme precision from the technique due to the practical aspects which are intended here and the obvious safety factors which have to be used in such a risky subject indeed, i.e., applications of physical techniques and ideas to financial data analysis for investment strategy purposes.

## 2. Detrended fluctuation analysis technique

The detrended fluctuation analysis (*DFA*) technique consists in dividing a time series or random one-variable sequence $y(t)$ of length $N$ into $N/\tau$ equal size nonoverlapping boxes [11,12]. The variable $t$ is discrete, evolves by a single unit at each time step between $t = 1$ and $N$. No data point is supposed to be missing. In other words, when applied to financial data, breaks due to holidays and week-ends are disregarded. Nevertheless, the $\tau$ units are said to be *days* in the following, a week has often 5 days, and a year about 250 days. Thus let each box contain $\tau$ points and $N/\tau$

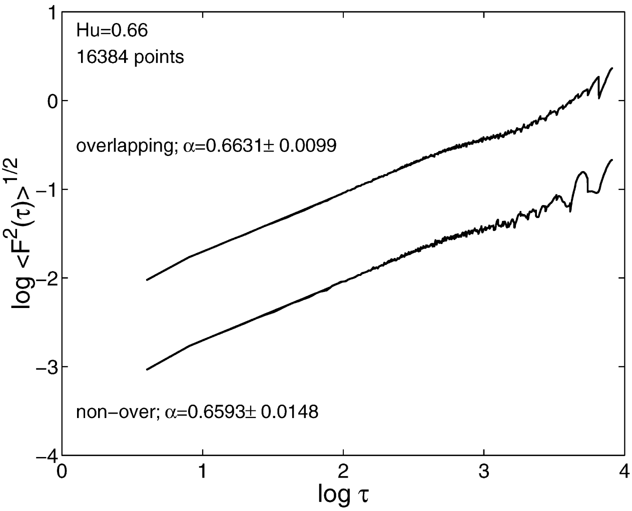


Fig. 1. Comparison of a 0.66 Hurst exponent fractional Brownian motion and the subsequent $\alpha$ value for a *DFA* technique when the $\tau$-size boxes are overlapping ($\alpha = 0.6631$) or not ($\alpha = 0.6593$).

be an integer. The local trend in each $\tau$-size box is assumed to be linear, i.e., it is taken as

$$z(t) = at + b\,. \tag{1}$$

In each $\tau$-size box one next calculates the root-mean-square deviation between $y(t)$ and $z(t)$. The detrended fluctuation function $F(\tau)$ is then calculated following

$$F^2(\tau) = \frac{1}{\tau} \sum_{t=k\tau+1}^{(k+1)\tau} |y(t) - z(t)|^2, \quad k = 0, 1, 2, \ldots, \left(\frac{N}{\tau} - 1\right)\,. \tag{2}$$

Averaging $F^2(\tau)$ over all $N/\tau$ box sizes centered on time $\tau$ gives the fluctuations $\langle F^2(\tau)\rangle$ as a function of $\tau$. The calculation is repeated for all possible different values of $\tau$. A power law behavior is expected as

$$\langle F^2(\tau)\rangle^{1/2} \sim \tau^{\alpha}\,. \tag{3}$$

An exponent $\alpha \neq \frac{1}{2}$ in a certain range of $\tau$ values implies the existence of long-range correlations in that time interval as in the fractional Brownian motion [13]. Such correlations are said to be "persistent" or "antipersistent" when they correspond to $\alpha > \frac{1}{2}$ and $\alpha < \frac{1}{2}$, respectively. The case of a fractional Brownian motion characterized by an input Hurst exponent [1] $Hu = 0.66$ is shown in Fig. 1; in practice $Hu = \alpha$. A straight line can well fit the data between $\log \tau = 1$ and 2.6. This interval is called the *scaling range*. As should be expected $\alpha \simeq 0.66$. Outside the scaling range the error

[1] A brief discussion of the Hurst exponent and other usual exponents found in the text is given in Appendix.

Table 1
Best fit exponent and scaling range of *FEXC* rate signals as obtained by the *DFA* technique (non-overlapping boxes)

| *FEXC* | $\alpha$ | *Scaling range* (Weeks) | *Crossover* |
|---|---|---|---|
| *USD/DEM* | 0.55 | 1–50 | — |
| *GBP/DEM* | 0.55 | 1–62 | — |
| *JPY/USD* | 0.55 | 1–101 | — |
| *USD/CAD* | 0.50 | 1–32 | — |
| *NLG/BEF* | 0.26 | 1–10 | *yes*[a] |
| *DEM/BEF* | 0.23 | 1–6 | *yes*[a] |
| *DKK/BEF* | 0.31 | 1–7 | *yes*[a] |
| *DEM/JPY* | 0.53 | 1–204 | — |
| *DEM/CHF* | 0.51 | 1–204 | — |
| *DEM/DKK* | 0.48 | 1–66 | — |
| *FRF/BEF* | 0.37 | 1–46 | — |
| *BLG/DEM* | 0.47 | 1–4 | — |
| *PLN/BEF* | 0.33 | 1–20 | — |

[a]"*yes*" indicates a crossover to Brownian motion ($\alpha = 0.50$) at large $\tau$.

bars are larger due to so-called finite size effects, and/or the lack of numerous data points.

Most of the time, for the foreign exchange currency (*FEXC*) rates that we have examined [14,15], the scaling range is well defined (Table 1). Sometimes it readily appears that the data contain two sets of points which can be fitted by straight lines. Usually that describing the "large $\tau$" data has a 0.50 slope. Such crossovers from fractional to ordinary Brownian motion are well observed. These crossovers suggest that correlated sequences have characteristic durations with well-defined lower and upper scales [14]. The specific $\alpha$ values can be thought to be related to political and economic conditions. The persistence is related to free market (and "runaway") conditions while the antipersistence develops due to strict political control allowing for a finite size bracket in which the *FEXC* rates can fluctuate. The case $\alpha = 0.50$ is surely avoided by speculators.

The case of overlapping boxes has been recently examined and the result shown in Fig. 1 for the above $Hu = 0.66$ case. To consider overlapping boxes might be useful when not many data points are available. However, it was feared that extra insidious correlations would thereby be inserted. Nevertheless, the analysis has shown that the value of $\alpha$ is rather insensitive to the way boxes are used. For example, in Fig. 1, the value of $\alpha$ is found to be ca. 0.66 whatever the overlapping condition, – the difference between values being quite small, i.e., about 0.6%.

A cubic trend, like

$$z(t) = ct^3 + dt^2 + et + f \,, \tag{4}$$

can be also considered [16]. The parameters $a$ to $f$ are similarly estimated through a best least-square fit of the data points in each box. Following the procedure described above the value of the exponent $\alpha$ can be obtained. Again the difference is found to be very small.

The interesting observation that coding and non-coding sequences in *DNA* could be sorted out by looking at a *local $\alpha$ value* [11,12] prompted us into observing whether a local $\alpha$ could be defined and searched for in the same way in financial data series [14–16]. This should allow one to probe the existence or not of *locally correlated or decorrelated sequences*. This has also been done for liquid water content and brightness temperature of stratus clouds [17]. In the *DNA* case, the $\alpha$ exponent drops below $\frac{1}{2}$ in the so-called non-coding regions. The exponent $\alpha$ jumps from much below $\frac{1}{2}$ to about $\frac{1}{2}$ when the clouds are breaking apart, and later drops back to a low value ... since there is no water or cloud surface temperature fluctuation anymore.

In order to observe the local correlations, a local observation box of a given finite size is constructed. Its size depends on the upper value of $\tau$ for which a reasonable power law exponent is found. It is chosen to be large enough in order to obtain a sufficiently large number of data points. The *local* exponent $\alpha$ is then calculated for the data contained in that finite size box, as above. Thereafter the box is moved along the time axis by an arbitrary finite number of points (say, corresponding to 20 days), depending on the intended strategy. The *local $\alpha$ exponent* can be displayed, e.g. for a Brownian motion long series as in Fig. 2. The *DFA* technique leads to an $\alpha$ value equal to 0.52, indicating the size of the error bar in such a procedure. The *local $\alpha$ exponent* in the displayed region varies between 0.47 and 0.55. Clearly, the *local $\alpha$ exponent* seems rougher, and varies with time around the overall (mean) $\alpha$ value. The variation depends on the box size. In Figs. 3–5, three typical *FEXC* rate time dependences, i.e., *DEM/JPY*, *DEM/CHF*, and *DEM/DKK* are shown for various time intervals. The $\alpha$ value is indicated with the scaling range. The *local* $\alpha$ value is shown as well for the latest years. For example, the *DEM/JPY local* $\alpha$ is consistently above 0.50 indicating a persistent evolution. A positive fluctuation is likely to be followed by another positive one. The case of *DEM/CHF* is typically Brownian with fluctuations around 0.50. However, in 1994 some drift is observed toward a value ca. 0.55 while in 1997–1998 some drift is observed toward a value ca. 0.45. It is clear that some economic or *FEXC* policy change occurred in 1994, and a drastic one at the end of 1998 in order to render the system more Brownian-like. The same is true for the *DEM/DKK* where, due to European economic policy, the spread in this exchange rate was changed several times, leading from a Brownian-like situation to a nowadays antipersistent behavior, i.e., a positive fluctuation is followed by a negative one, and conversely, such that *local* $\alpha$ is becoming pretty low these days.

Therefore, it can be claimed that this procedure interestingly leads to a *local* measurement of the degree of long-range correlations, thus of *local persistence* or not. In previous investigations [14], it has been found on thorough investigations of the *GBL/DEM* exchange rate data that the change in slope of *local* $\alpha$ vs. time corresponded to changes in the Bundesbank interest rate increase or decrease. However mere military events or political ones or others, like the Gulf War or Spanish nurses strikes, were somewhat irrelevant. The 1985 Plaza agreement though had some influence in

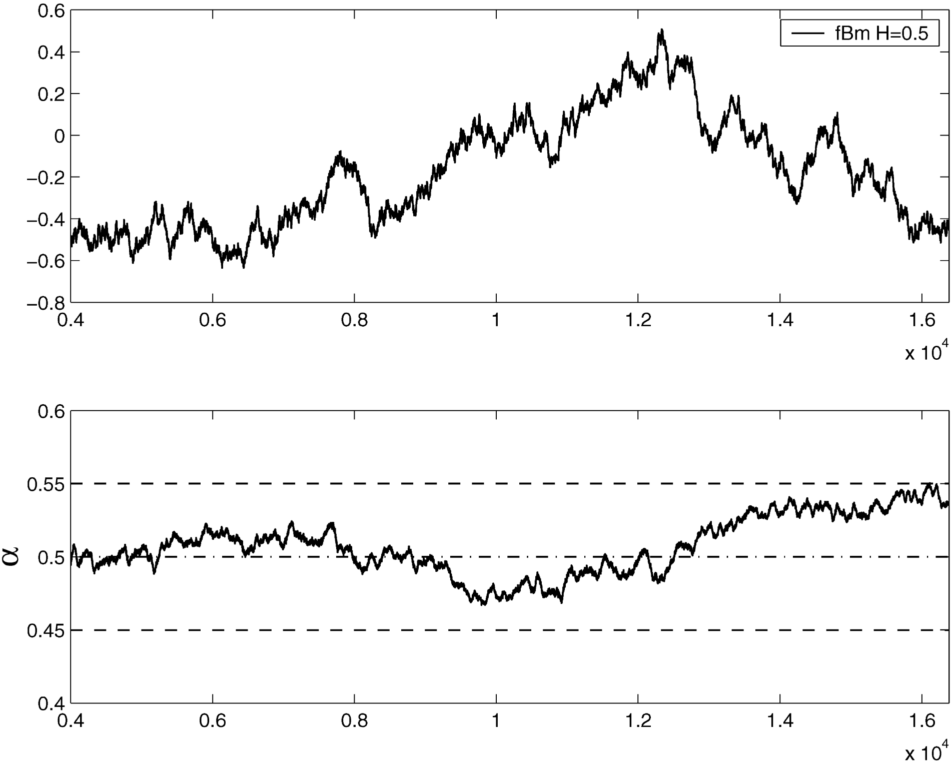


Fig. 2. The *DFA* technique result for the *local* $\alpha$ *exponent* of a (fractional) Brownian motion signal ($Hu=0.5$) in a restricted region of a long (16 834 data points) series following the method not taking into account the overlap of analyzing boxes; box size: 400 points.

order to curb the runaway *local* $\alpha$ value from a persistent 0.60 back to a more 0.50 Brownian-like value. Thus, *FEXC* rate behaviors indicate as for *DNA* and clouds that *local* $\alpha$ is a measure of information, an entropy variation indicating how (whatever) "information" is managed by the system, how fractional Brownian motion fluctuation processes stabilize (or not) a system. This seems to be an "information" to be taken into account when developing Hamiltonian or thermodynamic-like models.

Since local correlations can be sorted out, a strategy for profit making can be developed. It is easily observed whether there is persistency or antipersistency in some exchange rate, – according to the local value of $\alpha$. Thus, some guess whether the next fluctuation should be positive or negative can be made. Therefore a buy or sell decision can be taken. In so doing and taking the example of (*DEM*/*BEF*), we performed some virtual game and implemented the most simple strategy [18]. It can be shown that this technique leads to predictability and therefore financial gains on foreign exchange currency and other markets, – at least in the limit of zero fee constraint.

Starting with one (normalized) unit, assuming no fee (= playing as a banker), we have calculated the immediate daily compounded return. The capital gain over a 16-year period for the *DEM*/*USD* and *DEM*/*BEF*, starting on January 01, 1980 till

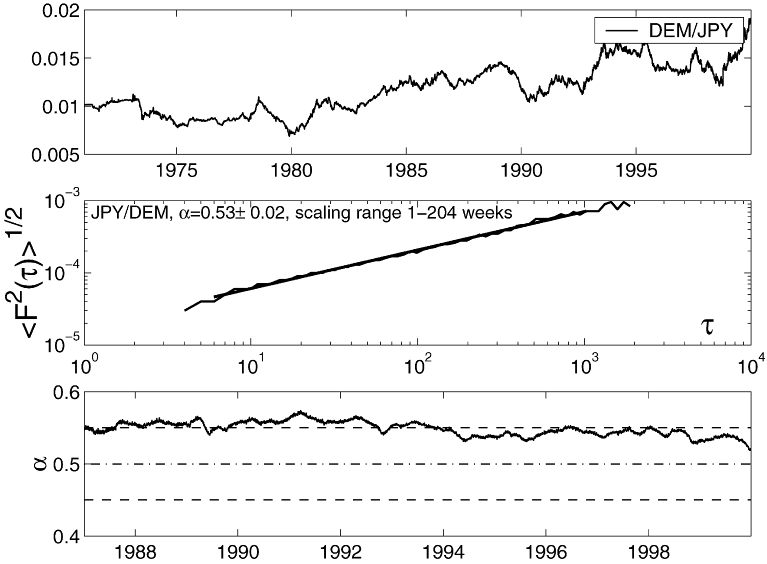


Fig. 3. Top of figure: variation of the *DEM/JPY* exchange rate from January 1, 1970 to December 31, 1999. Central part: Hurst exponent or $\alpha$ value from the *DFA* technique when the $\tau$-size boxes are not overlapping ($\alpha = 0.53$); scaling range: 1–204 weeks; $\tau$ in days. Bottom of figure: The *DFA local* $\alpha$ *exponent* technique result (non-overlapping boxes) for the latest thirteen years.

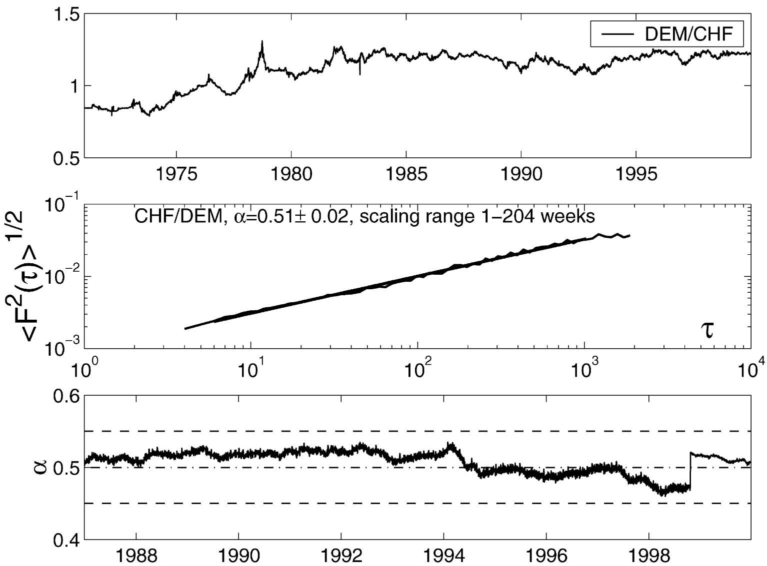


Fig. 4. Top of figure: variation of the *DEM/CHF* exchange rate from January 01, 1970 till December 31, 1999. Central part: Hurst exponent or $\alpha$ value from the *DFA* technique when the $\tau$-size boxes are not overlapping ($\alpha = 0.51$); scaling range: 1–204 weeks; $\tau$ in days. Bottom of figure: The *DFA local* $\alpha$ *exponent* technique result (non-overlapping boxes) for the latest thirteen years.

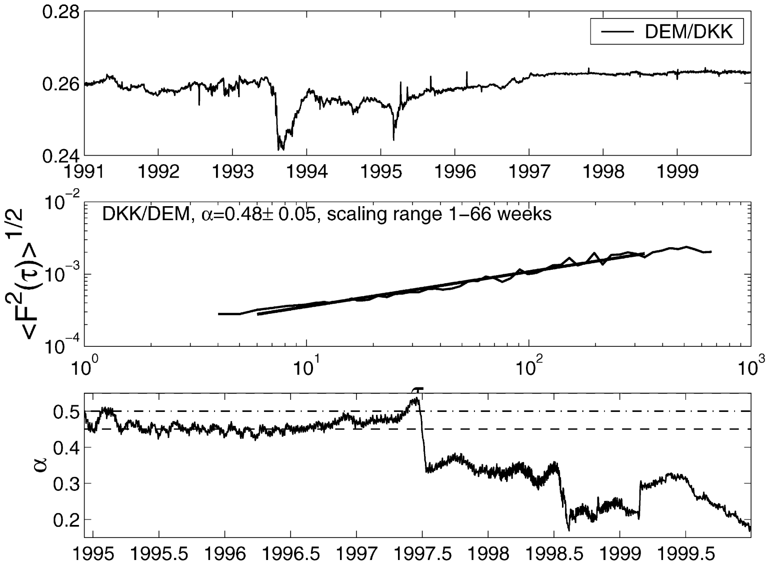


Fig. 5. Top of figure: variation of the *DEM/DKK* exchange rate from January 1, 1991 to December 31, 1999 (incorrectly labelled inset). Central part: Hurst exponent or $\alpha$ value for the same *DEM/DKK* (incorrectly labelled inset) time interval from the *DFA* technique when the $\tau$-size boxes are not overlapping ($\alpha = 0.48$); scaling range: 1−66 weeks; $\tau$ in days. Bottom of figure: The *DFA* local $\alpha$ exponent technique result (non-overlapping boxes) for the latest five years. Note: 0.265 *DEM* $\simeq$ 1.00 *DKK* on January 01, 2000.

October 15, 1996 has been calculated thus for a $N = 4200$ set of data points [18], taking as data input the value of the rate at closure in London. When *local* $\alpha$ was close to $\frac{1}{2}$ no guess was made, i.e., for 2210 data points. All guesses on the "next daily fluctuation" were correct. It is of interest to know that bad/good guesses were in the ratio 756/1234.

## 3. Zipf analysis technique

The above (*DFA*) technique considers the sign of the fluctuations and their persistence, but it falls short of implementing some strategy taking into account the *amplitude* of the fluctuations. Another type of analysis bringing information on coherence is based on the so-called Zipf analysis as applied to signals or "texts" [19]. This technique, originally introduced in the context of natural languages is performed by calculating the frequency of occurrence $f$ of each word in a given text. By sorting out the words according to their frequency, a rank $R$ can be assigned to each word, with $R = 1$ for the most frequent one. A power law

$$f \sim R^{-\zeta} \tag{5}$$

is expected [19] to be due to the hierarchical structure of the text as well as the presence of long-range correlations (sentences, and logical structures therein).

Table 2
Number of possible words of length $m$ for an alphabet with $k$ characters

| $m \setminus k$ | 2 | 3 | 4 |
|---|---|---|---|
| 2 | 4 | 9 | 16 |
| 3 | 8 | 27 | 64 |
| 4 | 16 | 51 | 256 |
| 5 | 32 | 153 | — |
| 6 | 61 | 459 | — |

A simple extension of the Zipf analysis is to consider $m$-letter words, i.e., the words strictly made of $m$ characters without considering the white spaces. The available number of different letters or characters $k$ in the alphabet should also be specified. The number of words for a given alphabet with a certain number of characters is given in Table 2. A power law in $f(R)$ is expected to be observed in correlated sequences [20,21]. There is no theory at this time predicting the exponent as a function of $(m,k)$.

It has been shown in previous work [20] that the technique is rather weak for a predictability purpose when only two characters and short words are considered [20]. Notice that in this approximation $(k=2)$ the signal is like a text, say written with d and u letters, translated into a $S=\pm 1$ Ising spin chain. An increase in the number of allowed characters for the alphabet allows one to consider different size and signs of the fluctuations, i.e., huge, marginal and small (positive or negative) fluctuations can be considered. The "too small" fluctuations, i.e., of weak interest for speculators, can be eliminated of consideration by introducing a threshold on the signal amplitude size.

After having decided on the number $(k)$ of characters (or Ising spin values) of the alphabet, the signal can be transformed into a text (or Ising chain), and thereafter analyzed with respect to the frequency of words of a given size $(m)$ to be ranked accordingly. In our work, words of equal lengths were always considered. The words can be ranked according to their frequency and a power law (or not) observed on a log–log plot.

The above procedure does not take into account the trend. For a positive (or negative) trend over the time box which is investigated, a bias can occur between words. For a two-character alphabet, e.g. u and d, the frequency $f$ of u's, i.e., $p_{\mathrm{u}}$ can be larger (smaller) than the frequency of d's, i.e., $p_{\mathrm{d}}$. Such a *bias* can be taken into account with respect to the equal probability occurrence, e.g. $\varepsilon = p_{\mathrm{u}} - 0.5 = p_{\mathrm{d}} + 0.5$. A new ranking procedure can be performed by defining the ratio of the observed frequency of a word divided by the theoretical frequency of occurrence of a word, assuming independence of characters. E.g. if the word uud occurs, say, $p_{\mathrm{uud}}$ times, since the independence of characters would imply that the word would occur $p_{\mathrm{u}}p_{\mathrm{u}}p_{\mathrm{d}}$ times, a relative frequency $f/f'$ can be defined as $p_{\mathrm{uud}}/(p_{\mathrm{u}}p_{\mathrm{u}}p_{\mathrm{d}})$. A new ranking can be made. It can be quite different (Table 3) from the previous one. In so doing a new power-law exponent $\zeta$ can be looked for.

A variant consists in comparing on a histogram [21] or on a normal–normal plot, the observed word occurrence probabilities vs. the theoretically expected ones, in absence

Table 3
Rank $R$ of words of size $m=4$ for a $k=2$ character alphabet for the Apple share value "signal" translated into a "text", taken at closing between January 01, 1987 and December 31, 1997[a]

| $R$ | $f$ | Text | $f'$ | $f/f'$ | $R'$ |
|---|---|---|---|---|---|
| 1 | 0.082478 | dddd | 0.092375 | 0.89286 | 15 |
| 2 | 0.080424 | dddu | 0.075183 | 1.0697 | 3 |
| 3 | 0.080424 | uddd | 0.075183 | 1.0697 | 3 |
| 4 | 0.076660 | dudd | 0.075183 | 1.0196 | 8 |
| 5 | 0.073580 | ddud | 0.075183 | 0.97868 | 10 |
| 6 | 0.070157 | dduu | 0.061191 | 1.1465 | 1 |
| 7 | 0.067077 | uudd | 0.061191 | 1.0962 | 2 |
| 8 | 0.063313 | uddu | 0.061191 | 1.0347 | 7 |
| 9 | 0.061259 | duud | 0.061191 | 1.0011 | 9 |
| 10 | 0.057837 | udud | 0.061191 | 0.94519 | 12 |
| 11 | 0.054757 | dudu | 0.061191 | 0.89485 | 14 |
| 12 | 0.052019 | uuud | 0.049803 | 1.0445 | 5 |
| 13 | 0.051677 | duuu | 0.049803 | 1.0376 | 6 |
| 14 | 0.045859 | uudu | 0.049803 | 0.92081 | 13 |
| 15 | 0.042779 | uduu | 0.049803 | 0.85896 | 16 |
| 16 | 0.038672 | uuuu | 0.040534 | 0.95406 | 11 |

[a] $f$ is the observed word frequency; $f'$ is the theoretical frequency taking into account the bias ($\varepsilon$) which is here $-0.0513$; due to the rescaling $f/f'$ the words take a new rank $R'$.

or in presence of trend. This has been implemented in order to show the *domino effect* at market crashes [22].

Another variant consists in ranking the words according to their relative frequency and relative ("normalized") rank taking into account for the normalization the probability $f_M$ of the most often occurring word [21]. Indeed for $m$ and $k$ large not all words do occur. Even though, e.g. for the ($m=6$, $k=2$) case there are 64 possible words, and the maximum rank is $R_M=64$, the frequency of the most often observed word is unknown. Thus a plot $f/f_M$ vs. $R_M/R$ will be different from $f$ vs. $R$, leading to a different $\zeta$ exponent.

In conclusion of this section, it is clear that different strategies following the Zipf analysis technique can be implemented, according to the following factors: the $(m,k)$ values, how the trend is eliminated (or not) and how the ranks and frequencies of occurrence are defined.

## 4. Moving-average analysis technique

Stock market indices have often appeared to be cyclical. In recent years, some less trivially deterministic content has been looked for as hidden in some characteristic noise parameter, e.g. the fractal dimension of the "signal". The fractal dimension $D$ is related to $Hu$, – see the appendix. We have discovered that a very usual technique used by chartists and analysts, known as the *moving-average analysis* (*MAA*) technique, contains an interesting way for determining $Hu$ for a $y(t)$ signal [23].

The moving average $\bar{y}$ is defined as

$$\bar{y}(t) = \frac{1}{N} \sum_{i=0}^{N-1} y(t-i)\,, \tag{6}$$

i.e., the average of $y$ for the last $N$ data points. It is recommended to consider the value of $\bar{y}$ as defined at the end of the examined interval. One can easily show that if $y$ increases (resp. decreases) with time, $\bar{y} < y$ (resp. $\bar{y} > y$). Thus, the moving average captures the trend of the signal over the considered time interval $N$.

Let two moving averages $\bar{y}_1$ and $\bar{y}_2$ be calculated, respectively, over e.g. $T_1$ and $T_2$ intervals such that $T_2 > T_1$. If $y(t)$ increases for a long period before decreasing rapidly, $\bar{y}_1$ will cross $\bar{y}_2$ from above. In empirical finance this event is called a *death* cross because the signal measured on a short-time interval decreases faster than the overall trend, as measure by the average in the longer interval [23]. This leads to pessimism concerning the behavior of the signal which should later hit some minimum. On the contrary, if $\bar{y}_1$ crosses $\bar{y}_2$ from below, the crossing point coincides with an upsurge of the signal $y(t)$, – such a crossing is called a *gold* cross by optimism; it occurs before a maximum. The density $\rho$ of crossing points between any two moving averages is obviously a measure of long-range power-law correlations in the signal.

It has been checked [24] that the density $\rho$ of crossing points between $\bar{y}_1$ and $\bar{y}_2$ curves is homogeneous along fractional Brownian motion signals, whence it is *not* a Cantor set [13]. In fact, the fractal dimension $D$ of the set of crossing points is trivially one. This suggests a lack of robustness, when forecast from *gold* and *death* cross distribution is made and this enables to work out the investing strategy.

However the density of crossing points $\rho(\Delta T)$, where $\Delta T = (T_2 - T_1)/T_2$, is fully symmetric, has a minimum [24] in the middle of the $\Delta T$ interval and diverges for $\Delta T = 0$ and for $\Delta T = 1$, with an exponent *which is the Hurst exponent*. This result certainly raises fundamental questions on the properties of (fractional or not) Brownian motion processes. The behavior of $\rho$ is analogous to the density of electronic states on a fractal lattice in a tight-binding approximation [25,26]. Notice that the moving-average method can serve *to measure the Hurst exponent* in a very fast, elegant and continuous way.

Investment strategies should look for *Hu* values over different time interval windows which are continuously shifted. In this respect, connection to multifractal analysis [2] can be made [29,30]. This corresponds to obtaining a spectrum of moving averages indeed [24]. Notice that a *DFA* and a multifractal analysis cost much more CPU time than a

[2] The technique consists in calculating the so-called "$q$th-order height–height correlation function" [27,28]

$$c_q(\tau) = \langle |y(t) - y(t')|^q \rangle_\tau\,, \tag{7}$$

where only non-zero terms are considered in the average $\langle \dots \rangle_\tau$ taken over all couples $(t, t')$ such that $\tau = |t - t'|$. The correlation function $c(\tau)$ is supposed to behave like

$$c_1(\tau) = \langle |y(t) - y(t')| \rangle_\tau \sim \tau^{H_1}\,, \tag{8}$$

where $H_1$ is the *Hu* exponent.

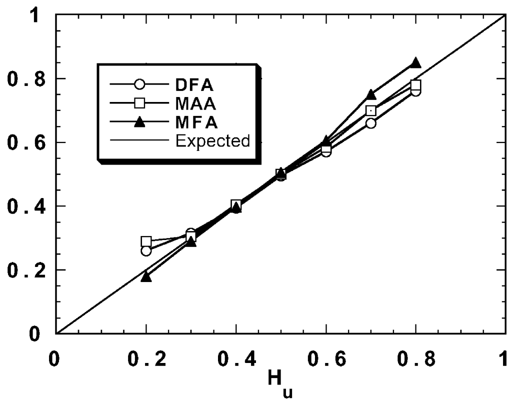


Fig. 6. The Hurst exponent for various fractional Brownian motion signals as measured by the detrended fluctuation analysis (*DFA*), moving average analysis (*MAA*) and multi-fractal analysis (*MFA*) methods.

moving-average method due to multiple loops which are present in the *DFA* and $c_q$ algorithms. The precision of the results is however quite similar, as shown in Fig. 6, gathering results from Refs. [14,24,29,31].

The technique has been used in order to analyze different signals [32]. It is recommended to look at results on the *http://www.supras.phys.ulg.ac.be/statphys/statphys.html* web site where colorful diagrams can be downloaded for different physical and financial time series.

## 5. Log-periodic financial index oscillation analysis technique

Among spectacular applications of statistical physics to the forecasting of stock markets is the finding of the possibility to predict crashes. This was first proposed in two independent works [33,34], though some authors are discontent with such a predictability [35,36]. It is proposed that the economic index $y(t)$ is similar to some thermodynamic property near a critical point, i.e.,

$$y(t) = A + B(t_c - t)^m[1 + C\cos(\omega \ln(t_c - t) + \phi)] \tag{9}$$

for $t < t_c$, where $t_c$ is the crash-time, rupture or critical point, $A, B, m, C, \omega, \phi$ are parameters. This index is finite at $t = t_c$ if $m > 0$, and it diverges if $m < 0$. On such laws, log-periodic oscillations are superposed [33,34,37–39]. The period of these oscillations converges to the rupture point as well. The law generalizes the scaleless situation of ordinary critical points to cases in which a discrete scale invariance [40] exists, i.e., to complex critical exponents of the form $m + i\omega$ [41]. This type of behavior was already proposed in order to fit experimental measurements of sound wave rate emissions prior to the rupture of heterogeneous composite stressed up to failure [42] and has been reported in biased diffusion on random lattices [43] and in sandpile avalanche studies on quasi-fractal bases [44].

It has appeared that the value of $m$ is not robust in the non-linear fits. Thus, we have proposed to consider a logarithmic divergence, corresponding to the $m=0$ limit, [37] rather than a power law, i.e.,

$$y(t) = A + B\ln\left(\frac{t_c - t}{t_c}\right)\left[1 + C\sin\left(\omega\ln\left(\frac{t_c - t}{t_c}\right) + \phi\right)\right] \quad \text{for } t < t_c\,. \tag{10}$$

In so doing the analysis of (closing value) stock market index like the Dow Jones industrial average (*DJIA*), the S&P 500 [37,38] and *DAX* [39] leads to observe the precursor of so-called crashes. This was shown on October 1987 and October 1997 cases, as it has been reported in the financial press in due time [45,46]. The prediction of the crash date was made as early as July, in the 1997 case. The error bar was subsequently reduced following further data acquisition, and the crash was predicted on October 24, 1997 to occur during the following week. It was remarkable that it occurred on the next open day, i.e., Monday, October 27, but the more so that the prediction could be made two months in advance.

The technique consists in fitting equally well the signal with the logarithmic law, with 3 free parameters, and to observe the departure from the general trend. The best is to do a non-linear fit, as for phase transition critical exponent search [1–5], and to search for the best estimates of the $A, B, t_c^{\mathrm{div}}$ [47]. An independent non-linear fit is made on the oscillations, and a second rupture point $t_c^{\mathrm{osc}}$ is estimated by selecting the successive maxima and the minima of the oscillations. The best parameters are obtained when an "identical" (within error bars) $t_c$ exists. It is best found from the relation

$$\frac{t_{n+1} - t_n}{t_n - t_{n-1}} = \frac{1}{\lambda}\,, \tag{11}$$

where $\lambda = \exp(\omega/2\pi)$ and $t_{n-1}$, $t_n$, $t_{n+1}$ are the successive converging maxima (resp. minima). After estimating $\lambda$ the rupture point $t_c^{\mathrm{osc}}$ is found from

$$t_c^{\mathrm{osc}} = \frac{t_n - t_{n+1}/\lambda}{1 - \frac{1}{\lambda}}\,. \tag{12}$$

Another more visual method consists in constructing the *envelope* of the index $y$: the upper envelope $y_{\max}$ and the lower one $y_{\min}$. The former represents the maximum of $y$ in an interval $[t_i, t]$ and the latter is the minimum of $y$ in an interval $[t, t_f]$. A pattern is observed to be made of a succession of peaks which seem to aggregate on each other at the crash time. It is recommended to look at results on the *http://www.supras.phys.ulg.ac.be/statphys/statphys.html* web site.

The parameter values have been published, i.e., the $\lambda$ and $t_c^{\mathrm{osc}}$ values obtained for the *DJIA* and S&P 500 are given in Table 1 of Ref. [37] for both 1980–1987 and 1990–1997 periods. We have stressed that the value of $\lambda$ seems to be universal in the sense that the rate of the convergence is always in the range 2.3–2.5. This seems to be related to a hidden (up to now) underlying (discrete scale) structure of the signal. In Ref. [48], the authors predicted $\omega = 2\pi/\ln\mu \sim 9.06$ for $\mu = 2$ value, for the density of states singularity of random system Hamiltonians. For the *DJIA* and S&P 500, our fits

[37] give an $\omega$ value of the order of [6.71,7.54]. This corresponds to a $\mu$ value $\sim 2.5$. Together with the spectral dimension $\sim 2(m+1)$, $\omega$ allows to determine the scaling factor of the number of degrees of freedom as well as the fractal dimension of the underlying space [48]. It can be conjectured that stock markets are hierarchical objects where each level has a different weight and a different characteristics time scale (the horizons of the investors). The hierarchical lattice might be a fractal tree [38] with loops. The geometry might control the type of criticality. This again seems relevant for implementing models and strategies and give new opportunities for physicists to study phase transitions on non-trivial lattices.

## 6. Conclusion

There is much data available in finance literature, from banks and markets, about costs, stocks and currency exchange rates or option values, on futures, discount and interest rates, ... the more so with the advent of the web. There are many levels of observation: individual income(s), individual expenses, checking accounts and savings, number of public or private accounts, volumes, debts and credits, tellers, dealers, bank outlets, businesses, governments, so many, that one is immediately tempted to play statistics. Fortunately, physicists have learned to develop intuition first during their studies and when doing some research. They can build models. However, these cannot be realistic if they do not reproduce experimental data. The above techniques have been described in order to give some insight into a few which can lead to fine analysis in order to obtain reliable values of characteristic parameters for physicists, those pertaining to the realm of power-law exponents. Later, physicists knowing the limits of their models and of their understanding will be honestly questioning at all levels their findings. At this time it is already possible to bring to Economy and in particular to Finance Theory a paraphernalia of tricks, theoretical or experimental ones.

Words like coherence effects, correlation lengths, relaxation times, many body interactions, grand canonical ensembles, spins, phase transitions, critical exponents, mean field approximations, renormalization group, cellular automata, organized criticality,.. are available for wrapping our gifts to business people. Moreover the techniques can be already implemented for personal investment, either with winning strategy or simply playing games with pocket money. Predictability models will not be found in a crystal ball but rather from models derived from spin glasses or sandpile avalanches and after using time-series analysis techniques as the ones here above.

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## Appendix

A warning is in order on the meaning of the above power-law exponents. Mathematically, they arise from specific mathematical tools and definitions, physically they represent different laws. It is not obvious that because two exponents can be related in the same way to, say, the fractal dimension or another that both exponents represent the same physical aspects of a signal. It seems pedagogically worthwhile to briefly review the most often used exponents and their meaning.

Both classic examples of a univariate stochastic time series are the Brownian motion (Bm) and the Lévy walk (Lw) cases [13]. In both cases, the power spectral density $S(f)$ of the time series has a power-law dependence on the frequency $f$ [49]:

$$S(f) \sim f^{-\beta} . \tag{A.1}$$

This formula allows one to put them into the self-affine class of phenomena. A crucial step is to examine the effect of noise in the data, i.e., to extract deterministic or stochastic components [50]. The stochastic aspects are found in the statistical distribution of values and its persistence. This is factually found to be either non-existent (white noise case) or if existent, to be strong or weak then. The correlations between events can be on a short or long range, and thereby so is the persistence strength. This is checked through the autocorrelation function. The larger the autocorrelation is at some $t$, the stronger is said to be the persistence, otherwise it is “weak”. The value of $t$ implies long- or short (time)-range persuasions, – with respect to the sampling frequency inverse. Moreover, the value of $\beta$ in Eq. (A.1) is valid only for a given range of the persistence in the time series. A Brownian motion is characterized by $\beta = 2$, and a white noise by $\beta = 0$.

Notice that the differences between adjacent values of a Brownian motion amplitude result in white noise. The Hurst $Hu$ exponent of a signal results from the “rescale range theory” (of Hurst [13,51,52]) and was first scientifically used to measure the Nile flooding and drought amplitudes. The Hurst method consists in listing the differences between the observed value at a discrete time $n$ over an interval with size $N$ on which the mean has been taken. The upper ($y_M$) and lower ($y_m$) values in that interval define the range $R_N = y_M - y_m$. The "rescale range" $R_N/S_N$ is expected to behave like $N^{Hu}$. This means that for a (discrete) self-affine signal $y(n)$, the neighborhood of a particular point on the signal can be rescaled by a factor $b$ using the roughness (or Hurst [53,54]) exponent $Hu$ and defining the new signal $b^{-Hu}y(bn)$. For the exponent value $Hu$, the frequency dependence of the signal so obtained should be undistinguishable from the original one, i.e.,

$$y(n) \sim b^{-Hu} y(bt) . \tag{A.2}$$

An exponent $Hu < \frac{1}{2}$ implies an *antipersistent* behavior while $Hu > \frac{1}{2}$ means a so-called *persistent* signal [13].

The simple Brownian motion is characterized by $Hu = \frac{1}{2}$ and white noise by $Hu = 0$ [13]. This seems to imply that

$$\beta = 2Hu - 1 . \tag{A.3}$$

Since a white noise is a truly random process, it can be concluded that $Hu = 0.5$ implies an uncorrelated time series, whence $Hu < 0.5$ implies antipersistence and $Hu > 0.5$ implies persistence. However, from preimposed $Hu$ values of a fractional Brownian motion series it is found that the equality (A.2) holds true in a very limited range and $\beta$ only slowly converges toward the value $Hu$.

The above results have been compared with those obtained from a detrended fluctuation analysis [14,16]. From the main text, it is expected that

$$\langle F^2 \rangle^{1/2} \sim t^{Ha} . \tag{A.4}$$

The exponent $Ha$ (called $\alpha$ in the main text) is officially called the Hausdorff exponent [13,54]. This exponent is sometimes called the Hurst exponent. This exponent is sometimes called the Holder exponent [55] as well. It is expected that

$$Ha = 2 - D , \tag{A.5}$$

where $D$ is the self-affine fractal dimension [13,56]. For Brownian motion, $Ha = 0.5$ and $D = 1.5$, while for white noise $Ha = 0$ and $D = 2$. A comparison of such exponents and their implication on the notion of "persistence" is given in Table 4.

The Zipf analysis as applied to signals or "texts" [19] as described here above leads to a power-law expectation

$$f \sim R^{-\zeta} . \tag{A.6}$$

This is due to the hierarchical structure of the text as well as the presence of long-range correlations (sentences, and logical structures therein). A conjecture

$$\zeta = |2Hu - 1| \tag{A.7}$$

has been proposed [57,58]. However, it seems that the relationship is not perfectly fulfilled. I conjecture here that the relationship holds if and only if the signal is stationary.

Table 4
Values of the most relevant exponents in various regimes (i.e., stationary, persistent, antipersistent) of univariate stochastic series[a]

| Name of the signal | $D$ | $Ha$ | $Hu$ | $\alpha$ | $\beta$ | $Ha$ |
|---|---|---|---|---|---|---|
| — | — | — | 0 | — | −1 | — |
| — | — | — | — | antipersist | — | station |
| — | — | — | 0.5 | 0.5 | 0 | uncorrrel |
| — | — | — | — | persist | — | station |
| WN | 2 | 0 | 1 | – | 1 | — |
| (f)Bm | — | — | — | superpers | — | nonstat |
| Bm | $\frac{3}{2}$ | 0.5 | $\frac{3}{2}$ | — | 2 | — |
| (f)Bm | — | 0 | 1 | — | 1 | — |
| flat | 1 | 1 | 2 | superpers | 3 | — |

[a] *D*: fractal dimension; *Ha*: Hausdorff measure; *Hu*: Hurst exponent; $\alpha$: from *DFA* technique; $\beta$: power spectrum exponent; WN=white noise; (f)Bm: (fractional) Brownian motion; flat: flat spectrum.

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